

## NAG Library Chapter Contents

### S – Approximations of Special Functions

S Chapter Introduction – a description of the Chapter and an overview of the algorithms available

Routine Name	Mark of Introduction	Purpose
s01baf	14	nagf_specfun_log_shifted $\ln(1 + x)$
s01eaf	14	nagf_specfun_exp_complex Complex exponential, $e^z$
s07aaf	1	nagf_specfun_tan $\tan x$
s09aaf	1	nagf_specfun_arcsin $\arcsin x$
s09abf	3	nagf_specfun_arccos $\arccos x$
s10aaf	3	nagf_specfun_tanh $\tanh x$
s10abf	4	nagf_specfun_sinh $\sinh x$
s10acf	4	nagf_specfun_cosh $\cosh x$
s11aaf	4	nagf_specfun_arctanh $\operatorname{arctanh} x$
s11abf	4	nagf_specfun_arcsinh $\operatorname{arcsinh} x$
s11acf	4	nagf_specfun_arccosh $\operatorname{arccosh} x$
s13aaf	1	nagf_specfun_integral_exp Exponential integral $E_1(x)$
s13acf	2	nagf_specfun_integral_cos Cosine integral $\operatorname{Ci}(x)$
s13adf	5	nagf_specfun_integral_sin Sine integral $\operatorname{Si}(x)$
s14aaf	1	nagf_specfun_gamma Gamma function
s14abf	8	nagf_specfun_gamma_log_real Log gamma function, real argument
s14acf	14	nagf_specfun_polygamma $\psi(x) - \ln x$
s14adf	14	nagf_specfun_polygamma_deriv Scaled derivatives of $\psi(x)$
s14aef	20	nagf_specfun_psi_deriv_real Polygamma function $\psi^{(n)}(x)$ for real $x$
s14aff	20	nagf_specfun_psi_deriv_complex Polygamma function $\psi^{(n)}(z)$ for complex $z$
s14agf	21	nagf_specfun_gamma_log_complex Logarithm of the gamma function $\ln \Gamma(z)$ , complex argument
s14ahf	23	nagf_specfun_gamma_log_scaled_real Scaled log gamma function
s14baf	14	nagf_specfun_gamma_incomplete Incomplete gamma functions $P(a, x)$ and $Q(a, x)$
s14cbf	24	nagf_specfun_beta_log_real Logarithm of the beta function $\ln B(a, b)$

s14ccf	24	nagf_specfun_beta_incomplete Incomplete beta function $I_x(a, b)$ and its complement $1 - I_x$
s15abf	3	nagf_specfun_cdf_normal Cumulative Normal distribution function $P(x)$
s15acf	4	nagf_specfun_compcdf_normal Complement of cumulative Normal distribution function $Q(x)$
s15adf	4	nagf_specfun_erfc_real Complement of error function $\text{erfc}(x)$
s15aef	4	nagf_specfun_erf_real Error function $\text{erf}(x)$
s15aff	7	nagf_specfun_dawson Dawson's integral
s15agf	22	nagf_specfun_erfcx_real Scaled complement of error function, $\text{erfcx}(x)$
s15ddf	14	nagf_specfun_erfc_complex Scaled complex complement of error function, $\exp(-z^2) \text{erfc}(-iz)$
s17acf	1	nagf_specfun_bessel_y0_real Bessel function $Y_0(x)$
s17adf	1	nagf_specfun_bessel_y1_real Bessel function $Y_1(x)$
s17aef	5	nagf_specfun_bessel_j0_real Bessel function $J_0(x)$
s17aff	5	nagf_specfun_bessel_j1_real Bessel function $J_1(x)$
s17agf	8	nagf_specfun_airy_ai_real Airy function $\text{Ai}(x)$
s17ahf	8	nagf_specfun_airy_bi_real Airy function $\text{Bi}(x)$
s17ajf	8	nagf_specfun_airy_ai_deriv Airy function $\text{Ai}'(x)$
s17akf	8	nagf_specfun_airy_bi_deriv Airy function $\text{Bi}'(x)$
s17alf	20	nagf_specfun_bessel_zeros Zeros of Bessel functions $J_\alpha(x)$ , $J'_\alpha(x)$ , $Y_\alpha(x)$ or $Y'_\alpha(x)$
s17aqf	24	nagf_specfun_bessel_y0_real_vector Bessel function vectorized $Y_0(x)$
s17arf	24	nagf_specfun_bessel_y1_real_vector Bessel function vectorized $Y_1(x)$
s17ASF	24	nagf_specfun_bessel_j0_real_vector Bessel function vectorized $J_0(x)$
s17atf	24	nagf_specfun_bessel_j1_real_vector Bessel function vectorized $J_1(x)$
s17auf	24	nagf_specfun_airy_ai_real_vector Airy function vectorized $\text{Ai}(x)$
s17avf	24	nagf_specfun_airy_bi_real_vector Airy function vectorized $\text{Bi}(x)$
s17awf	24	nagf_specfun_airy_ai_deriv_vector Derivatives of the Airy function, vectorized $\text{Ai}'(x)$
s17axf	24	nagf_specfun_airy_bi_deriv_vector Derivatives of the Airy function, vectorized $\text{Bi}'(x)$
s17dcf	13	nagf_specfun_bessel_y_complex Bessel functions $Y_{\nu+a}(z)$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$
s17def	13	nagf_specfun_bessel_j_complex Bessel functions $J_{\nu+a}(z)$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$
s17dgf	13	nagf_specfun_airy_ai_complex Airy functions $\text{Ai}(z)$ and $\text{Ai}'(z)$ , complex $z$
s17dhf	13	nagf_specfun_airy_bi_complex Airy functions $\text{Bi}(z)$ and $\text{Bi}'(z)$ , complex $z$

s17dlf	13	nagf_specfun_hankel_complex Hankel functions $H_{\nu+a}^{(j)}(z)$ , $j = 1, 2$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$
s17gaf	26.1	nagf_specfun_struve_h0 Struve function of order 0, $H_0(x)$
s17gbf	26.1	nagf_specfun_struve_h1 Struve function of order 1, $H_1(x)$
s18acf	1	nagf_specfun_bessel_k0_real Modified Bessel function $K_0(x)$
s18adf	1	nagf_specfun_bessel_k1_real Modified Bessel function $K_1(x)$
s18aef	5	nagf_specfun_bessel_i0_real Modified Bessel function $I_0(x)$
s18aff	5	nagf_specfun_bessel_i1_real Modified Bessel function $I_1(x)$
s18aqf	24	nagf_specfun_bessel_k0_real_vector Modified Bessel function vectorized $K_0(x)$
s18arf	24	nagf_specfun_bessel_k1_real_vector Modified Bessel function vectorized $K_1(x)$
s18ASF	24	nagf_specfun_bessel_i0_real_vector Modified Bessel function vectorized $I_0(x)$
s18atf	24	nagf_specfun_bessel_i1_real_vector Modified Bessel function vectorized $I_1(x)$
s18ccf	10	nagf_specfun_bessel_k0_scaled Scaled modified Bessel function $e^x K_0(x)$
s18cdf	10	nagf_specfun_bessel_k1_scaled Scaled modified Bessel function $e^x K_1(x)$
s18cef	10	nagf_specfun_bessel_i0_scaled Scaled modified Bessel function $e^{- x } I_0(x)$
s18cff	10	nagf_specfun_bessel_i1_scaled Scaled modified Bessel function $e^{- x } I_1(x)$
s18cqf	24	nagf_specfun_bessel_k0_scaled_vector Scaled modified Bessel function vectorized $e^x K_0(x)$
s18crf	24	nagf_specfun_bessel_k1_scaled_vector Scaled modified Bessel function vectorized $e^x K_1(x)$
s18csf	24	nagf_specfun_bessel_i0_scaled_vector Scaled modified Bessel function vectorized $e^{- x } I_0(x)$
s18ctf	24	nagf_specfun_bessel_i1_scaled_vector Scaled modified Bessel function vectorized $e^{- x } I_1(x)$
s18dcf	13	nagf_specfun_bessel_k_complex Modified Bessel functions $K_{\nu+a}(z)$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$
s18def	13	nagf_specfun_bessel_i_complex Modified Bessel functions $I_{\nu+a}(z)$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$
s18gaf	26.1	nagf_specfun_struve_l0 Modified Struve function of order 0, $L_0(x)$
s18gbf	26.1	nagf_specfun_struve_l1 Modified Struve function of order 1, $L_1(x)$
s18gcf	26.1	nagf_specfun_struve_i0ml0 The function $I_0(x) - L_0(x)$ , where $I_0(x)$ is a modified Bessel function and $L_0(x)$ is a Struve function
s18gdf	26.1	nagf_specfun_struve_i1ml1 The function $I_1(x) - L_1(x)$ , where $I_1(x)$ is a modified Bessel function and $L_1(x)$ is a Struve function
s18gkf	21	nagf_specfun_bessel_j_seq_complex Bessel function of the 1st kind $J_{\alpha \pm n}(z)$
s19aab	11	nagf_specfun_kelvin_ber Kelvin function $\text{ber } x$
s19abf	11	nagf_specfun_kelvin bei Kelvin function $\text{bei } x$

s19acf	11	nagf_specfun_kelvin_ker Kelvin function ker $x$
s19adf	11	nagf_specfun_kelvin_kei Kelvin function kei $x$
s19anf	24	nagf_specfun_kelvin_ber_vector Kelvin function vectorized ber $x$
s19apf	24	nagf_specfun_kelvin_bei_vector Kelvin function vectorized bei $x$
s19aqf	24	nagf_specfun_kelvin_ker_vector Kelvin function vectorized ker $x$
s19arf	24	nagf_specfun_kelvin_kei_vector Kelvin function vectorized kei $x$
s20acf	5	nagf_specfun_fresnel_s Fresnel integral $S(x)$
s20adf	5	nagf_specfun_fresnel_c Fresnel integral $C(x)$
s20aqf	24	nagf_specfun_fresnel_s_vector Fresnel integral vectorized $S(x)$
s20arf	24	nagf_specfun_fresnel_c_vector Fresnel integral vectorized $C(x)$
s21baf	8	nagf_specfun_ellipint_symm_1_degen Degenerate symmetrised elliptic integral of 1st kind $R_C(x, y)$
s21bbf	8	nagf_specfun_ellipint_symm_1 Symmetrised elliptic integral of 1st kind $R_F(x, y, z)$
s21bcf	8	nagf_specfun_ellipint_symm_2 Symmetrised elliptic integral of 2nd kind $R_D(x, y, z)$
s21bdf	8	nagf_specfun_ellipint_symm_3 Symmetrised elliptic integral of 3rd kind $R_J(x, y, z, r)$
s21bef	22	nagf_specfun_ellipint_legendre_1 Elliptic integral of 1st kind, Legendre form, $F(\phi \mid m)$
s21bff	22	nagf_specfun_ellipint_legendre_2 Elliptic integral of 2nd kind, Legendre form, $E(\phi \mid m)$
s21bgf	22	nagf_specfun_ellipint_legendre_3 Elliptic integral of 3rd kind, Legendre form, $\Pi(n; \phi \mid m)$
s21bhf	22	nagf_specfun_ellipint_complete_1 Complete elliptic integral of 1st kind, Legendre form, $K(m)$
s21bjf	22	nagf_specfun_ellipint_complete_2 Complete elliptic integral of 2nd kind, Legendre form, $E(m)$
s21caf	15	nagf_specfun_jacellip_real Jacobi elliptic functions sn, cn and dn of real argument
s21cbf	20	nagf_specfun_jacellip_complex Jacobi elliptic functions sn, cn and dn of complex argument
s21ccf	20	nagf_specfun_jactheta_real Jacobi theta functions $\theta_k(x, q)$ of real argument
s21daf	20	nagf_specfun_ellipint_general_2 General elliptic integral of 2nd kind $F(z, k', a, b)$ of complex argument
s22aa	20	nagf_specfun_legendre_p Legendre functions of 1st kind $P_n^m(x)$ or $\overline{P_n^m}(x)$
s22baf	24	nagf_specfun_1fl_real Real confluent hypergeometric function ${}_1F_1(a; b; x)$
s22bbf	24	nagf_specfun_1fl_real_scaled Real confluent hypergeometric function ${}_1F_1(a; b; x)$ in scaled form
s22bef	25	nagf_specfun_2fl_real Real Gauss hypergeometric function ${}_2F_1(a, b; c; x)$
s22bff	25	nagf_specfun_2fl_real_scaled Real Gauss hypergeometric function ${}_2F_1(a, b; c; x)$ in scaled form
s30aab	22	nagf_specfun_opt_bsm_price Black–Scholes–Merton option pricing formula

s30abf	22	nagf_specfun_opt_bsm_greeks Black–Scholes–Merton option pricing formula with Greeks
s30baf	22	nagf_specfun_opt_lookback_fls_price Floating-strike lookback option pricing formula in the Black-Scholes-Merton model
s30bbf	22	nagf_specfun_opt_lookback_fls_greeks Floating-strike lookback option pricing formula with Greeks in the Black-Scholes-Merton model
s30caf	22	nagf_specfun_opt_binary_con_price Binary option, cash-or-nothing pricing formula
s30cbf	22	nagf_specfun_opt_binary_con_greeks Binary option, cash-or-nothing pricing formula with Greeks
s30ccf	22	nagf_specfun_opt_binary_aon_price Binary option, asset-or-nothing pricing formula
s30cdf	22	nagf_specfun_opt_binary_aon_greeks Binary option, asset-or-nothing pricing formula with Greeks
s30faf	22	nagf_specfun_opt_barrier_std_price Standard barrier option pricing formula
s30jaf	22	nagf_specfun_opt_jumpdiff_merton_price Jump-diffusion, Merton's model, option pricing formula
s30jbf	22	nagf_specfun_opt_jumpdiff_merton_greeks Jump-diffusion, Merton's model, option pricing formula with Greeks
s30naf	22	nagf_specfun_opt_heston_price Heston's model option pricing formula
s30nbf	23	nagf_specfun_opt_heston_greeks Heston's model option pricing formula with Greeks
s30ncf	25	nagf_specfun_opt_heston_term Heston's model option pricing with term structure
s30qcf	22	nagf_specfun_opt_amer_bs_price American option, Bjerksund and Stensland pricing formula
s30saf	22	nagf_specfun_opt_asian_geom_price Asian option, geometric continuous average rate pricing formula
s30sbf	22	nagf_specfun_opt_asian_geom_greeks Asian option, geometric continuous average rate pricing formula with Greeks

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