

# NAG Library Routine Document

## F07AFF (DGEEQU)

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

### 1 Purpose

F07AFF (DGEEQU) computes diagonal scaling matrices  $D_R$  and  $D_C$  intended to equilibrate a real  $m$  by  $n$  matrix  $A$  and reduce its condition number.

### 2 Specification

```
SUBROUTINE F07AFF (M, N, A, LDA, R, C, ROWCND, COLCND, AMAX, INFO)
  INTEGER          M, N, LDA, INFO
  REAL (KIND=nag_wp) A(LDA,*), R(M), C(N), ROWCND, COLCND, AMAX
```

The routine may be called by its LAPACK name *dgeequ*.

### 3 Description

F07AFF (DGEEQU) computes the diagonal scaling matrices. The diagonal scaling matrices are chosen to try to make the elements of largest absolute value in each row and column of the matrix  $B$  given by

$$B = D_R A D_C$$

have absolute value 1. The diagonal elements of  $D_R$  and  $D_C$  are restricted to lie in the safe range  $(\delta, 1/\delta)$ , where  $\delta$  is the value returned by routine X02AMF. Use of these scaling factors is not guaranteed to reduce the condition number of  $A$  but works well in practice.

### 4 References

None.

### 5 Arguments

- |    |   |              |
|----|---|--------------|
| 1: | M – INTEGER   | <i>Input</i> |
|    | <i>On entry:</i> $m$ , the number of rows of the matrix $A$ .   |              |
|    | <i>Constraint:</i> $M \geq 0$ .   |              |
| 2: | N – INTEGER   | <i>Input</i> |
|    | <i>On entry:</i> $n$ , the number of columns of the matrix $A$ .  |              |
|    | <i>Constraint:</i> $N \geq 0$ .   |              |
| 3: | A(LDA,*) – REAL (KIND=nag_wp) array   | <i>Input</i> |
|    | <b>Note:</b> the second dimension of the array $A$ must be at least $\max(1, N)$ .  |              |
|    | <i>On entry:</i> the matrix $A$ whose scaling factors are to be computed.   |              |
| 4: | LDA – INTEGER   | <i>Input</i> |
|    | <i>On entry:</i> the first dimension of the array $A$ as declared in the (sub)program from which F07AFF (DGEEQU) is called. |              |
|    | <i>Constraint:</i> $LDA \geq \max(1, M)$ .  |              |

- 5: R(M) – REAL (KIND=nag\_wp) array Output  
*On exit:* if INFO = 0 or INFO > M, R contains the row scale factors, the diagonal elements of  $D_R$ . The elements of R will be positive.
- 6: C(N) – REAL (KIND=nag\_wp) array Output  
*On exit:* if INFO = 0, C contains the column scale factors, the diagonal elements of  $D_C$ . The elements of C will be positive.
- 7: ROWCND – REAL (KIND=nag\_wp) Output  
*On exit:* if INFO = 0 or INFO > M, ROWCND contains the ratio of the smallest value of  $R(i)$  to the largest value of  $R(i)$ . If ROWCND  $\geq$  0.1 and AMAX is neither too large nor too small, it is not worth scaling by  $D_R$ .
- 8: COLCND – REAL (KIND=nag\_wp) Output  
*On exit:* if INFO = 0, COLCND contains the ratio of the smallest value of  $C(i)$  to the largest value of  $C(i)$ .  
 If COLCND  $\geq$  0.1, it is not worth scaling by  $D_C$ .
- 9: AMAX – REAL (KIND=nag\_wp) Output  
*On exit:*  $\max |a_{ij}|$ . If AMAX is very close to overflow or underflow, the matrix  $A$  should be scaled.
- 10: INFO – INTEGER Output  
*On exit:* INFO = 0 unless the routine detects an error (see Section 6).

## 6 Error Indicators and Warnings

INFO < 0

If INFO =  $-i$ , argument  $i$  had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0 and INFO  $\leq$  M

Row  $\langle value \rangle$  of  $A$  is exactly zero.

INFO > M

Column  $\langle value \rangle$  of  $A$  is exactly zero.

## 7 Accuracy

The computed scale factors will be close to the exact scale factors.

## 8 Parallelism and Performance

F07AFF (DGEEQU) is not threaded in any implementation.

## 9 Further Comments

The complex analogue of this routine is F07ATF (ZGEEQU).

## 10 Example

This example equilibrates the general matrix  $A$  given by

$$A = \begin{pmatrix} 1.80 \times 10^{10} & 2.88 \times 10^{10} & 2.05 & -8.90 \times 10^9 \\ 5.25 & -2.95 & -9.50 \times 10^{-9} & -3.80 \\ 1.58 & -2.69 & -2.90 \times 10^{-10} & -1.04 \\ -1.11 & -0.66 & -5.90 \times 10^{-11} & 0.80 \end{pmatrix}.$$

Details of the scaling factors, and the scaled matrix are output.

### 10.1 Program Text

Program f07affe

```
!      F07AFF Example Program Text
!
!      Mark 26 Release. NAG Copyright 2016.
!
!      .. Use Statements ..
!      Use nag_library, Only: dgeequ, dscal, f06fcf, nag_wp, x02ajf, x02amf,      &
!                               x02bhf, x04caf
!
!      .. Implicit None Statement ..
!      Implicit None
!
!      .. Parameters ..
!      Real (Kind=nag_wp), Parameter      :: one = 1.0_nag_wp
!      Real (Kind=nag_wp), Parameter      :: thresh = 0.1_nag_wp
!      Integer, Parameter                  :: nin = 5, nout = 6
!
!      .. Local Scalars ..
!      Real (Kind=nag_wp)                  :: amax, big, colcnd, rowcnd, small
!      Integer                              :: i, ifail, info, j, lda, n
!
!      .. Local Arrays ..
!      Real (Kind=nag_wp), Allocatable     :: a(:,,:), c(:), r(:)
!
!      .. Intrinsic Procedures ..
!      Intrinsic                           :: real
!
!      .. Executable Statements ..
!      Write (nout,*) 'F07AFF Example Program Results'
!      Write (nout,*)
!      Flush (nout)
!
!      Skip heading in data file
!      Read (nin,*)
!      Read (nin,*) n
!      lda = n
!      Allocate (a(lda,n),c(n),r(n))
!
!      Read the N by N matrix A from data file
!
!      Read (nin,*)(a(i,1:n),i=1,n)
!
!      Print the matrix A
!
!      ifail: behaviour on error exit
!      =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
!      ifail = 0
!      Call x04caf('General',' ',n,n,a,lda,'Matrix A',ifail)
!      Write (nout,*)
!
!      Compute row and column scaling factors
!
!      The NAG name equivalent of dgeequ is f07aff
!      Call dgeequ(n,n,a,lda,r,c,rowcnd,colcnd,amax,info)
!
!      If (info>0) Then
!        If (info<=n) Then
!          Write (nout,99999) 'Row ', info, ' of A is exactly zero'
!        Else
!          Write (nout,99999) 'Column ', info - n, ' of A is exactly zero'
!        End If
!      Else
```

```

!      Print ROWCND, COLCND, AMAX and the scale factors

      Write (nout,99998) 'ROWCND =', rowcnd, ', COLCND =', colcnd,
      ' ', AMAX =', amax
      Write (nout,*)
      Write (nout,*) 'Row scale factors'
      Write (nout,99997) r(1:n)
      Write (nout,*)
      Write (nout,*) 'Column scale factors'
      Write (nout,99997) c(1:n)
      Write (nout,*)
      Flush (nout)

!      Compute values close to underflow and overflow

      small = x02amf()/(x02ajf()*real(x02bhf(),kind=nag_wp))
      big = one/small
      If ((rowcnd>=thresh) .And. (amax>=small) .And. (amax<=big)) Then
        If (colcnd<thresh) Then

!          Just column scale A
!          The NAG name equivalent of dscal is f06edf
          Do j = 1, n
            Call dscal(n,c(j),a(1,j),1)
          End Do

          End If
        Else If (colcnd>=thresh) Then

!          Just row scale A
          Do j = 1, n
            Call f06fcf(n,r,1,a(1,j),1)
          End Do

          Else

!          Row and column scale A
          Do j = 1, n
            Call dscal(n,c(j),a(1,j),1)
            Call f06fcf(n,r,1,a(1,j),1)
          End Do

          End If

!          Print the scaled matrix
          ifail = 0
          Call x04caf('General',' ',n,n,a,lda,'Scaled matrix',ifail)

          End If

99999 Format (1X,A,I4,A)
99998 Format (1X,3(A,1P,E8.1))
99997 Format ((1X,1P,7E11.2))
      End Program f07affe

```

## 10.2 Program Data

F07AFF Example Program Data

```

4                                     :Value of N

1.80D+10  2.88D+10  2.05D+00  -8.90D+09
5.25D+00  -2.95D+00  -9.50D-09  -3.80D+00
1.58D+00  -2.69D+00  -2.90D-10  -1.04D+00
-1.11D+00  -6.60D-01  -5.90D-11  8.00D-01 :End of matrix A

```

### 10.3 Program Results

F07AFF Example Program Results

Matrix A

	1	2	3	4
1	1.8000E+10	2.8800E+10	2.0500E+00	-8.9000E+09
2	5.2500E+00	-2.9500E+00	-9.5000E-09	-3.8000E+00
3	1.5800E+00	-2.6900E+00	-2.9000E-10	-1.0400E+00
4	-1.1100E+00	-6.6000E-01	-5.9000E-11	8.0000E-01

ROWCND = 3.9E-11, COLCND = 1.8E-09, AMAX = 2.9E+10

Row scale factors

3.47E-11	1.90E-01	3.72E-01	9.01E-01
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Column scale factors

1.00E+00	1.00E+00	5.53E+08	1.38E+00
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Scaled matrix

	1	2	3	4
1	0.6250	1.0000	0.0393	-0.4269
2	1.0000	-0.5619	-1.0000	-1.0000
3	0.5874	-1.0000	-0.0596	-0.5341
4	-1.0000	-0.5946	-0.0294	0.9957

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