

## NAG Library Function Document

### nag\_legendre\_p (s22aac)

#### 1 Purpose

nag\_legendre\_p (s22aac) returns a sequence of values for either the unnormalized or normalized Legendre functions of the first kind  $P_n^m(x)$  or  $\overline{P}_n^m(x)$  for real  $x$  of a given order  $m$  and degree  $n = 0, 1, \dots, N$ .

#### 2 Specification

```
#include <nag.h>
#include <nags.h>

void nag_legendre_p (Integer mode, double x, Integer m, Integer nl,
                    double p[], NagError *fail)
```

#### 3 Description

nag\_legendre\_p (s22aac) evaluates a sequence of values for either the unnormalized or normalized Legendre ( $m = 0$ ) or associated Legendre ( $m \neq 0$ ) functions of the first kind  $P_n^m(x)$  or  $\overline{P}_n^m(x)$ , where  $x$  is real with  $-1 \leq x \leq 1$ , of order  $m$  and degree  $n = 0, 1, \dots, N$  defined by

$$P_n^m(x) = (1-x^2)^{m/2} \frac{d^m}{dx^m} P_n(x) \quad \text{if } m \geq 0,$$

$$P_n^m(x) = \frac{(n+m)!}{(n-m)!} P_n^{-m}(x) \quad \text{if } m < 0 \quad \text{and}$$

$$\overline{P}_n^m(x) = \sqrt{\frac{(2n+1)(n-m)!}{2(n+m)!}} P_n^m(x)$$

respectively;  $P_n(x)$  is the (unassociated) Legendre polynomial of degree  $n$  given by

$$P_n(x) \equiv P_n^0(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$$

(the *Rodrigues formula*). Note that some authors (e.g., Abramowitz and Stegun (1972)) include an additional factor of  $(-1)^m$  (the *Condon–Shortley Phase*) in the definitions of  $P_n^m(x)$  and  $\overline{P}_n^m(x)$ . They use the notation  $P_{mn}(x) \equiv (-1)^m P_n^m(x)$  in order to distinguish between the two cases.

nag\_legendre\_p (s22aac) is based on a standard recurrence relation described in Section 8.5.3 of Abramowitz and Stegun (1972). Constraints are placed on the values of  $m$  and  $n$  in order to avoid the possibility of machine overflow. It also sets the appropriate elements of the array **p** (see Section 5) to zero whenever the required function is not defined for certain values of  $m$  and  $n$  (e.g.,  $m = -5$  and  $n = 3$ ).

#### 4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

## 5 Arguments

- 1: **mode** – Integer *Input*  
*On entry:* indicates whether the sequence of function values is to be returned unnormalized or normalized.  
**mode** = 1  
 The sequence of function values is returned unnormalized.  
**mode** = 2  
 The sequence of function values is returned normalized.  
*Constraint:* **mode** = 1 or 2.
- 2: **x** – double *Input*  
*On entry:* the argument  $x$  of the function.  
*Constraint:*  $\text{abs}(\mathbf{x}) \leq 1.0$ .
- 3: **m** – Integer *Input*  
*On entry:* the order  $m$  of the function.  
*Constraint:*  $\text{abs}(\mathbf{m}) \leq 27$ .
- 4: **nl** – Integer *Input*  
*On entry:* the degree  $N$  of the last function required in the sequence.  
*Constraints:*  
 $\mathbf{nl} \geq 0$ ;  
 if  $\mathbf{m} = 0$ ,  $\mathbf{nl} \leq 100$ ;  
 if  $\mathbf{m} \neq 0$ ,  $\mathbf{nl} \leq 55 - \text{abs}(\mathbf{m})$ .
- 5: **p[**nl** + 1]** – double *Output*  
*On exit:* the required sequence of function values as follows:  
 if **mode** = 1, **p**[ $n$ ] contains  $F_n^m(x)$ , for  $n = 0, 1, \dots, N$ ;  
 if **mode** = 2, **p**[ $n$ ] contains  $\overline{F}_n^m(x)$ , for  $n = 0, 1, \dots, N$ .
- 6: **fail** – NagError \* *Input/Output*  
 The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 3.2.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle \text{value} \rangle$  had an illegal value.

### NE\_INT

On entry,  $|\mathbf{m}| = \langle \text{value} \rangle$ .

Constraint:  $|\mathbf{m}| \leq 27$ .

On entry, **mode** =  $\langle value \rangle$ .

Constraint: **mode**  $\leq 2$ .

On entry, **mode** =  $\langle value \rangle$ .

Constraint: **mode**  $\geq 1$ .

On entry, **nl** =  $\langle value \rangle$ .

Constraint: **nl**  $\geq 0$ .

## NE\_INT\_2

On entry, **nl** =  $\langle value \rangle$  and **m** =  $\langle value \rangle$ .

Constraint: **nl** + **m**  $\leq 55$ .

On entry, **nl** =  $\langle value \rangle$ .

Constraint: **nl**  $\leq 100$  when **m** = 0.

## NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 3.6.6 in How to Use the NAG Library and its Documentation for further information.

## NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 3.6.5 in How to Use the NAG Library and its Documentation for further information.

## NE\_REAL

On entry, **x** =  $\langle value \rangle$ .

Constraint: **x**  $\leq 1.0$ .

## 7 Accuracy

The computed function values should be accurate to within a small multiple of the *machine precision* except when underflow (or overflow) occurs, in which case the true function values are within a small multiple of the underflow (or overflow) threshold of the machine.

## 8 Parallelism and Performance

nag\_legendre\_p (s22aac) is not threaded in any implementation.

## 9 Further Comments

None.

## 10 Example

This example reads the values of the arguments  $x$ ,  $m$  and  $N$  from a file, calculates the sequence of unnormalized associated Legendre function values  $P_n^m(x)$ ,  $P_{n+1}^m(x)$ ,  $\dots$ ,  $P_{n+N}^m(x)$ , and prints the results.

### 10.1 Program Text

```
/* nag_legendre_p (s22aac) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * NAG C Library
```

```

*
* Mark 26, 2016.
*
*/

#include <stdio.h>
#include <string.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer exit_status = 0, m, mode, n, nl;
    NagError fail;
    char *str = 0;
    double *p = 0, x;

    INIT_FAIL(fail);

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
    printf("nag_legendre_p (s22aac) Example Program Results\n");
#ifdef _WIN32
    scanf_s("%" NAG_IFMT " %lf %" NAG_IFMT " %" NAG_IFMT "", &mode, &x, &m,
            &nl);
#else
    scanf("%" NAG_IFMT " %lf %" NAG_IFMT " %" NAG_IFMT "", &mode, &x, &m, &nl);
#endif
    if (!(p = NAG_ALLOC(nl + 1, double)) || !(str = NAG_ALLOC(80, char)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
    if (mode == 1) {
        if (m == 0)
#ifdef _WIN32
            strcpy_s(str, 80, "Unnormalized Legendre function values\n");
#else
            strcpy(str, "Unnormalized Legendre function values\n");
#endif
        else
#ifdef _WIN32
            strcpy_s(str, 80, "Unnormalized associated Legendre function values\n");
#else
            strcpy(str, "Unnormalized associated Legendre function values\n");
#endif
    }
    else if (mode == 2) {
        if (m == 0)
#ifdef _WIN32
            strcpy_s(str, 80, "Normalized Legendre function values\n");
#else
            strcpy(str, "Normalized Legendre function values\n");
#endif
        else
#ifdef _WIN32
            strcpy_s(str, 80, "Normalized associated Legendre function values\n");
#else
            strcpy(str, "Normalized associated Legendre function values\n");
#endif
    }

    /* nag_legendre_p (s22aac).
     * Legendre and associated Legendre functions of the first
     * kind with real arguments

```

```

*/
nag_legendre_p(mode, x, m, nl, p, &fail);
printf("mode      x      m      nl\n");
printf("%3" NAG_IFMT "      %5.1f%6" NAG_IFMT "%6" NAG_IFMT "\n\n", mode, x,
      m, nl);

if (fail.code == NE_NOERROR) {
    printf("%s\n", str);
    printf(" n      P(n)\n");
    for (n = 0; n <= nl; ++n)
        printf("%2" NAG_IFMT " %13.4e\n", n, p[n]);
}
else {
    printf("Error from nag_legendre_p (s22aac).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
END:
NAG_FREE(p);
NAG_FREE(str);
return exit_status;
}

```

## 10.2 Program Data

nag\_legendre\_p (s22aac) Example Program Data  
 1 0.5 2 3 : Values of mode, x, m and nl

## 10.3 Program Results

nag\_legendre\_p (s22aac) Example Program Results  
 mode x m nl  
 1 0.5 2 3

Unnormalized associated Legendre function values

n	P(n)
0	0.0000e+00
1	0.0000e+00
2	2.2500e+00
3	5.6250e+00

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