NAG Library Function Document

nag_arcsinh (s11abc)

1 Purpose

nag_arcsinh (s11abc) returns the value of the inverse hyperbolic sine, $\operatorname{arcsinh} x$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_arcsinh (double x)
```

3 Description

nag_arcsinh (s11abc) calculates an approximate value for the inverse hyperbolic sine of its argument, $\operatorname{arcsinh} x$.

For $|x| \leq 1$ it is based on the Chebyshev expansion

$$\operatorname{arcsinh} x = x \times y(t) = x \sum_{r=0} c_r T_r(t), \qquad \text{where } t = 2x^2 - 1.$$

For $|x| > 1$ it uses the fact that

$$\operatorname{arcsinh} x = \operatorname{sign} x \times \ln\left(|x| + \sqrt{x^2 + 1}\right).$$

This form is used directly for $1 < |x| < 10^k$, where $k = n/2 + 1$, and the machine uses approximately n decimal place arithmetic.

For $|x| \geq 10^k$, $\sqrt{x^2 + 1}$ is equal to $|x|$ to within the accuracy of the machine and hence we can guard against premature overflow and, without loss of accuracy, calculate

$$\operatorname{arcsinh} x = \operatorname{sign} x \times (\ln 2 + \ln|x|).$$

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: **x** – double *Input*

On entry: the argument x of the function.

6 Error Indicators and Warnings

None.

7 Accuracy

If δ and ϵ are the relative errors in the argument and the result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{\sqrt{1+x^2}\,\mathrm{arcsinh}\,x}\delta \right|.$$

That is, the relative error in the argument, x, is amplified by a factor at least $\frac{x}{\sqrt{1+x^2}\,\mathrm{arcsinh}\,x}$, in the result.

The equality should hold if δ is greater than the ***machine precision*** (δ due to data errors etc.) but if δ is simply due to round-off in the machine representation it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the amplification factor is shown in the following graph:

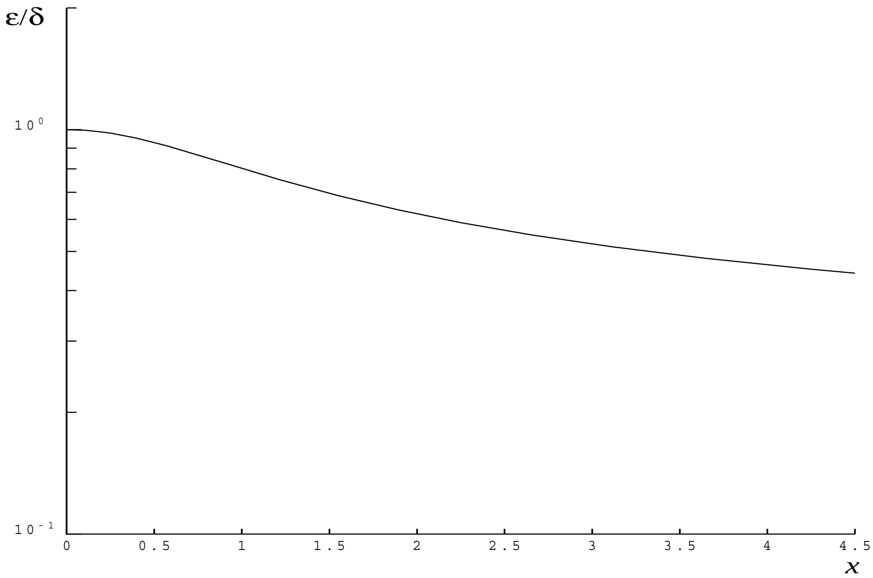

Figure 1

It should be noted that this factor is always less than or equal to one. For large x we have the absolute error in the result, E, in principle, given by

$$E \sim \delta.$$

This means that eventually accuracy is limited by ***machine precision***.

8 Parallelism and Performance

nag_arcsinh (s11abc) is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
/* nag_arcsinh (s11abc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Integer exit_status = 0;
  double x, y;
  NagError fail;

  INIT_FAIL(fail);

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]s");
#else
  scanf("%*[^\n]s");
#endif
  printf("nag_arcsinh (s11abc) Example Program Results\n");
  printf("     x           y\n");
#ifdef _WIN32
  while (scanf_s("%lf", &x) != EOF)
#else
  while (scanf("%lf", &x) != EOF)
#endif
  {
    /* nag_arcsinh (s11abc).
     * Inverse hyperbolic sine, arcsinh x
     */
    y = nag_arcsinh(x);
    if (fail.code != NE_NOERROR) {
      printf("Error from nag_arcsinh (s11abc).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
    printf("%12.3e%12.3e\n", x, y);
  }

END:
  return exit_status;
}
```

10.2 Program Data

```
nag_arcsinh (s11abc) Example Program Data
            -2.0
            -0.5
             1.0
             6.0
```

10.3 Program Results

```
nag_arcsinh (s11abc) Example Program Results
     x          y
  -2.000e+00  -1.444e+00
  -5.000e-01  -4.812e-01
   1.000e+00   8.814e-01
   6.000e+00   2.492e+00
```