

NAG Library Function Document

nag_shifted_log (s01bac)

1 Purpose

nag_shifted_log (s01bac) returns a value of the shifted logarithmic function, $\ln(1+x)$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_shifted_log (double x, NagError *fail)
```

3 Description

nag_shifted_log (s01bac) computes values of $\ln(1+x)$, retaining full relative precision even when $|x|$ is small. The function is based on the Chebyshev expansion

$$\ln \frac{1+p^2+2p\bar{x}}{1+p^2-2p\bar{x}} = 4 \sum_{k=0}^{\infty} \frac{p^{2k+1}}{2k+1} T_{2k+1}(\bar{x}).$$

Setting $\bar{x} = \frac{x(1+p^2)}{2p(x+2)}$, and choosing $p = \frac{q-1}{q+1}$, $q = \sqrt[4]{2}$ the expansion is valid in the domain $x \in \left[\frac{1}{\sqrt{2}} - 1, \sqrt{2} - 1 \right]$.

Outside this domain, $\ln(1+x)$ is computed by the standard logarithmic function.

4 References

Lyusternik L A, Chervonenkis O A and Yanpolskii A R (1965) *Handbook for Computing Elementary Functions* p. 57 Pergamon Press

5 Arguments

- 1: **x** – double *Input*
On entry: the argument x of the function.
Constraint: $x > -1.0$.
- 2: **fail** – NagError * *Input/Output*
 The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_REAL_ARG_LE

On entry, $x = \langle \text{value} \rangle$.
 Constraint: $x > -1.0$.

7 Accuracy

The returned result should be accurate almost to *machine precision*, with a limit of about 20 significant figures due to the precision of internal constants. Note however that if x lies very close to -1.0 and is not exact (for example if x is the result of some previous computation and has been rounded), then precision will be lost in the computation of $1 + x$, and hence $\ln(1 + x)$, in `nag_shifted_log` (s01bac).

8 Parallelism and Performance

`nag_shifted_log` (s01bac) is not threaded in any implementation.

9 Further Comments

Empirical tests show that the time taken for a call of `nag_shifted_log` (s01bac) usually lies between about 1.25 and 2.5 times the time for a call to the standard logarithm function.

10 Example

The example program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```

/* nag_shifted_log (s01bac) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer exit_status = 0;
    double x, y;
    NagError fail;

    INIT_FAIL(fail);

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
    printf("nag_shifted_log (s01bac) Example Program Results\n");
    printf("      x              y\n");
#ifdef _WIN32
    while (scanf_s("%lf", &x) != EOF)
#else
    while (scanf("%lf", &x) != EOF)
#endif
    {
        /* nag_shifted_log (s01bac).
         * ln(1+x)
         */
        y = nag_shifted_log(x, &fail);
        if (fail.code != NE_NOERROR) {
            printf("Error from nag_shifted_log (s01bac).\n%s\n", fail.message);
        }
    }
}

```

```
        exit_status = 1;
        goto END;
    }
    printf("%13.4e %13.4e\n", x, y);
}

END:
    return exit_status;
}
```

10.2 Program Data

```
nag_shifted_log (s01bac) Example Program Data
 2.50e+0
 1.25e-1
-9.06e-1
 1.29e-3
-7.83e-6
 1.00e-9
```

10.3 Program Results

```
nag_shifted_log (s01bac) Example Program Results
      x              y
 2.5000e+00    1.2528e+00
 1.2500e-01    1.1778e-01
-9.0600e-01   -2.3645e+00
 1.2900e-03    1.2892e-03
-7.8300e-06   -7.8300e-06
 1.0000e-09    1.0000e-09
```
