

## NAG Library Function Document

### nag\_2\_sample\_t\_test (g07cac)

#### 1 Purpose

nag\_2\_sample\_t\_test (g07cac) computes a  $t$ -test statistic to test for a difference in means between two Normal populations, together with a confidence interval for the difference between the means.

#### 2 Specification

```
#include <nag.h>
#include <nagg07.h>

void nag_2_sample_t_test (Nag_TailProbability tail, Nag_PopVar equal,
    Integer nx, Integer ny, double xmean, double ymean, double xstd,
    double ystd, double clevel, double *t, double *df, double *prob,
    double *dl, double *du, NagError *fail)
```

#### 3 Description

Consider two independent samples, denoted by  $X$  and  $Y$ , of size  $n_x$  and  $n_y$  drawn from two Normal populations with means  $\mu_x$  and  $\mu_y$ , and variances  $\sigma_x^2$  and  $\sigma_y^2$  respectively. Denote the sample means by  $\bar{x}$  and  $\bar{y}$  and the sample variances by  $s_x^2$  and  $s_y^2$  respectively.

nag\_2\_sample\_t\_test (g07cac) calculates a test statistic and its significance level to test the null hypothesis  $H_0 : \mu_x = \mu_y$ , together with upper and lower confidence limits for  $\mu_x - \mu_y$ . The test used depends on whether or not the two population variances are assumed to be equal.

1. It is assumed that the two variances are equal, that is  $\sigma_x^2 = \sigma_y^2$ .

The test used is the two sample  $t$ -test. The test statistic  $t$  is defined by;

$$t_{\text{obs}} = \frac{\bar{x} - \bar{y}}{s \sqrt{(1/n_x) + (1/n_y)}}$$

where  $s^2 = \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2}$  is the pooled variance of the two samples.

Under the null hypothesis  $H_0$  this test statistic has a  $t$ -distribution with  $(n_x + n_y - 2)$  degrees of freedom.

The test of  $H_0$  is carried out against one of three possible alternatives:

- (i)  $H_1 : \mu_x \neq \mu_y$ ; the significance level,  $p = P(t \geq |t_{\text{obs}}|)$ , i.e., a two tailed probability.
- (ii)  $H_1 : \mu_x > \mu_y$ ; the significance level,  $p = P(t \geq t_{\text{obs}})$ , i.e., an upper tail probability.
- (iii)  $H_1 : \mu_x < \mu_y$ ; the significance level,  $p = P(t \leq t_{\text{obs}})$ , i.e., a lower tail probability.

Upper and lower  $100(1 - \alpha)\%$  confidence limits for  $\mu_x - \mu_y$  are calculated as:

$$(\bar{x} - \bar{y}) \pm t_{1-\alpha/2} s \sqrt{(1/n_x) + (1/n_y)},$$

where  $t_{1-\alpha/2}$  is the  $100(1 - \alpha/2)$  percentage point of the  $t$ -distribution with  $(n_x + n_y - 2)$  degrees of freedom.

2. It is not assumed that the two variances are equal.

If the population variances are not equal the usual two sample  $t$ -statistic no longer has a  $t$ -distribution and an approximate test is used.

This problem is often referred to as the Behrens–Fisher problem, see Kendall and Stuart (1979). The test used here is based on Satterthwaites procedure. To test the null hypothesis the test statistic  $t'$  is used where

$$t'_{\text{obs}} = \frac{\bar{x} - \bar{y}}{\text{se}(\bar{x} - \bar{y})}$$

where  $\text{se}(\bar{x} - \bar{y}) = \sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}$ .

A  $t$ -distribution with  $f$  degrees of freedom is used to approximate the distribution of  $t'$  where

$$f = \frac{\text{se}(\bar{x} - \bar{y})^4}{\frac{s_x^2/n_x^2}{(n_x - 1)} + \frac{s_y^2/n_y^2}{(n_y - 1)}}$$

The test of  $H_0$  is carried out against one of the three alternative hypotheses described above, replacing  $t$  by  $t'$  and  $t_{\text{obs}}$  by  $t'_{\text{obs}}$ .

Upper and lower  $100(1 - \alpha)\%$  confidence limits for  $\mu_x - \mu_y$  are calculated as:

$$(\bar{x} - \bar{y}) \pm t_{1-\alpha/2} \text{se}(x - \bar{y}).$$

where  $t_{1-\alpha/2}$  is the  $100(1 - \alpha/2)$  percentage point of the  $t$ -distribution with  $f$  degrees of freedom.

## 4 References

Johnson M G and Kotz A (1969) *The Encyclopedia of Statistics 2* Griffin

Kendall M G and Stuart A (1979) *The Advanced Theory of Statistics (3 Volumes)* (4th Edition) Griffin

Snedecor G W and Cochran W G (1967) *Statistical Methods* Iowa State University Press

## 5 Arguments

1: **tail** – Nag\_TailProbability *Input*

*On entry:* indicates which tail probability is to be calculated, and thus which alternative hypothesis is to be used.

**tail** = Nag\_TwoTail

The two tail probability, i.e.,  $H_1 : \mu_x \neq \mu_y$ .

**tail** = Nag\_UpperTail

The upper tail probability, i.e.,  $H_1 : \mu_x > \mu_y$ .

**tail** = Nag\_LowerTail

The lower tail probability, i.e.,  $H_1 : \mu_x < \mu_y$ .

*Constraint:* **tail** = Nag\_UpperTail, Nag\_LowerTail or Nag\_TwoTail.

2: **equal** – Nag\_PopVar *Input*

*On entry:* indicates whether the population variances are assumed to be equal or not.

**equal** = Nag\_PopVarEqual

The population variances are assumed to be equal, that is  $\sigma_x^2 = \sigma_y^2$ .

**equal** = Nag\_PopVarNotEqual

The population variances are not assumed to be equal.

*Constraint:* **equal** = Nag\_PopVarEqual or Nag\_PopVarNotEqual.

- 3: **nx** – Integer *Input*  
*On entry:* the size of the  $X$  sample,  $n_x$ .  
*Constraint:* **nx**  $\geq 2$ .
- 4: **ny** – Integer *Input*  
*On entry:* the size of the  $Y$  sample,  $n_y$ .  
*Constraint:* **ny**  $\geq 2$ .
- 5: **xmean** – double *Input*  
*On entry:* the mean of the  $X$  sample,  $\bar{x}$ .
- 6: **ymean** – double *Input*  
*On entry:* the mean of the  $Y$  sample,  $\bar{y}$ .
- 7: **xstd** – double *Input*  
*On entry:* the standard deviation of the  $X$  sample,  $s_x$ .  
*Constraint:* **xstd**  $> 0.0$ .
- 8: **ystd** – double *Input*  
*On entry:* the standard deviation of the  $Y$  sample,  $s_y$ .  
*Constraint:* **ystd**  $> 0.0$ .
- 9: **clevel** – double *Input*  
*On entry:* the confidence level,  $1 - \alpha$ , for the specified tail. For example **clevel** = 0.95 will give a 95% confidence interval.  
*Constraint:*  $0.0 < \mathbf{clevel} < 1.0$ .
- 10: **t** – double \* *Output*  
*On exit:* contains the test statistic,  $t_{\text{obs}}$  or  $t'_{\text{obs}}$ .
- 11: **df** – double \* *Output*  
*On exit:* contains the degrees of freedom for the test statistic.
- 12: **prob** – double \* *Output*  
*On exit:* contains the significance level, that is the tail probability,  $p$ , as defined by **tail**.
- 13: **dl** – double \* *Output*  
*On exit:* contains the lower confidence limit for  $\mu_x - \mu_y$ .
- 14: **du** – double \* *Output*  
*On exit:* contains the upper confidence limit for  $\mu_x - \mu_y$ .
- 15: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_BAD\_PARAM

On entry, argument **equal** had an illegal value.

On entry, argument **tail** had an illegal value.

### NE\_INT\_ARG\_LT

On entry, **nx** =  $\langle value \rangle$ .

Constraint: **nx**  $\geq 2$ .

On entry, **ny** =  $\langle value \rangle$ .

Constraint: **ny**  $\geq 2$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

### NE\_REAL\_ARG\_GE

On entry, **clevel** must not be greater than or equal to 1.0: **clevel** =  $\langle value \rangle$ .

### NE\_REAL\_ARG\_LE

On entry, **clevel** must not be less than or equal to 0.0: **clevel** =  $\langle value \rangle$ .

On entry, **xstd** must not be less than or equal to 0.0: **xstd** =  $\langle value \rangle$ .

On entry, **ystd** must not be less than or equal to 0.0: **ystd** =  $\langle value \rangle$ .

## 7 Accuracy

The computed probability and the confidence limits should be accurate to approximately five significant figures.

## 8 Parallelism and Performance

nag\_2\_sample\_t\_test (g07cac) is not threaded in any implementation.

## 9 Further Comments

The sample means and standard deviations can be computed using nag\_summary\_stats\_onevar (g01atc).

## 10 Example

The following example program reads the two sample sizes and the sample means and standard deviations for two independent samples. The data is taken from page 116 of Snedecor and Cochran (1967) from a test to compare two methods of estimating the concentration of a chemical in a vat. A test of the equality of the means is carried out first assuming that the two population variances are equal and then making no assumption about the equality of the population variances.

### 10.1 Program Text

```
/* nag_2_sample_t_test (g07cac) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
```

```

*
*/

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg07.h>

int main(void)
{
    /* Local variables */
    double prob, xstd, ystd;
    double t;
    double xmean, ymean, df, dl, du;
    double clevel;

    Integer exit_status = 0, nx, ny;
    NagError fail;

    INIT_FAIL(fail);

    printf("nag_2_sample_t_test (g07cac) Example Program Results\n");
    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT " %" NAG_IFMT "", &nx, &ny);
#else
    scanf("%" NAG_IFMT " %" NAG_IFMT "", &nx, &ny);
#endif
#ifdef _WIN32
    scanf_s("%lf %lf %lf %lf", &xmean, &ymean, &xstd, &ystd);
#else
    scanf("%lf %lf %lf %lf", &xmean, &ymean, &xstd, &ystd);
#endif
#ifdef _WIN32
    scanf_s("%lf", &clevel);
#else
    scanf("%lf", &clevel);
#endif

    /* nag_2_sample_t_test (g07cac).
     * Computes t-test statistic for a difference in means
     * between two Normal populations, confidence interval
     */
    nag_2_sample_t_test(Nag_TwoTail, Nag_PopVarEqual, nx, ny, xmean, ymean,
                       xstd, ystd, clevel, &t, &df, &prob, &dl, &du, &fail);

    if (fail.code != NE_NOERROR) {
        printf("Error from nag_2_sample_t_test (g07cac).\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }
    printf("\nAssuming population variances are equal.\n\n");
    printf("t test statistic = %10.4f\n", t);
    printf("Degrees of freedom = %8.1f\n", df);
    printf("Significance level = %8.4f\n", prob);
    printf("Lower confidence limit for difference in means = %10.4f\n", dl);
    printf("Upper confidence limit for difference in means = %10.4f\n", du);

    /* nag_2_sample_t_test (g07cac), see above. */
    nag_2_sample_t_test(Nag_TwoTail, Nag_PopVarNotEqual, nx, ny, xmean, ymean,
                       xstd, ystd, clevel, &t, &df, &prob, &dl, &du, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_2_sample_t_test (g07cac).\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }
}

```

```
printf("\nNo assumptions about population variances.\n\n");
printf("t test statistic = %10.4f\n", t);
printf("Degrees of freedom = %8.4f\n", df);
printf("Significance level = %8.4f\n", prob);
printf("Lower confidence limit for difference in means = %10.4f\n", dl);
printf("Upper confidence limit for difference in means = %10.4f\n", du);
END:
return exit_status;
}
```

## 10.2 Program Data

```
nag_2_sample_t_test (g07cac) Example Program Data
4 8
25.0 21.0
0.8185 4.2083
0.95
```

## 10.3 Program Results

```
nag_2_sample_t_test (g07cac) Example Program Results
```

Assuming population variances are equal.

```
t test statistic =      1.8403
Degrees of freedom =      10.0
Significance level =    0.0955
Lower confidence limit for difference in means =    -0.8429
Upper confidence limit for difference in means =     8.8429
```

No assumptions about population variances.

```
t test statistic =      2.5922
Degrees of freedom =      7.9925
Significance level =    0.0320
Lower confidence limit for difference in means =     0.4410
Upper confidence limit for difference in means =     7.5590
```

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