

## NAG Library Function Document

### nag\_regsn\_mult\_linear\_tran\_model (g02dkc)

#### 1 Purpose

nag\_regsn\_mult\_linear\_tran\_model (g02dkc) calculates the estimates of the arguments of a general linear regression model for given constraints from the singular value decomposition results.

#### 2 Specification

```
#include <nag.h>
#include <nagg02.h>

void nag_regsn_mult_linear_tran_model (Integer ip, Integer iconst,
    const double p[], const double c[], Integer tdc, double b[], double rss,
    double df, double se[], double cov[], NagError *fail)
```

#### 3 Description

nag\_regsn\_mult\_linear\_tran\_model (g02dkc) computes the estimates given a set of linear constraints for a general linear regression model which is not of full rank. It is intended for use after a call to nag\_regsn\_mult\_linear (g02dac) or nag\_regsn\_mult\_linear\_upd\_model (g02ddc).

In the case of a model not of full rank the functions use a singular value decomposition (SVD) to find the parameter estimates,  $\hat{\beta}_{svd}$ , and their variance-covariance matrix. Details of the SVD are made available, in the form of the matrix  $P^*$ :

$$P^* = \begin{pmatrix} D^{-1}P_1^T \\ P_0^T \end{pmatrix}$$

as described by nag\_regsn\_mult\_linear (g02dac) and nag\_regsn\_mult\_linear\_upd\_model (g02ddc).

Alternative solutions can be formed by imposing constraints on the arguments. If there are  $p$  arguments and the rank of the model is  $k$ , then  $n_c = p - k$  constraints will have to be imposed to obtain a unique solution.

Let  $C$  be a  $p$  by  $n_c$  matrix of constraints, such that

$$C^T\beta = 0,$$

then the new parameter estimates  $\hat{\beta}_c$  are given by:

$$\begin{aligned} \hat{\beta}_c &= A\hat{\beta}_{svd} \\ &= \left( I - P_0(C^TP_0)^{-1} \right) \hat{\beta}_{svd}, \end{aligned}$$

where  $I$  is the identity matrix, and the variance-covariance matrix is given by:

$$AP_1D^{-2}P_1^TA^T$$

provided  $(C^TP_0)^{-1}$  exists.

## 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Hammarling S (1985) The singular value decomposition in multivariate statistics *SIGNUM Newsl.* **20(3)** 2–25

Searle S R (1971) *Linear Models* Wiley

## 5 Arguments

- 1: **ip** – Integer *Input*  
*On entry:* the number of terms in the linear model,  $p$ .  
*Constraint:*  $\mathbf{ip} \geq 1$ .
  
- 2: **iconst** – Integer *Input*  
*On entry:* the number of constraints to be imposed on the arguments,  $n_c$ .  
*Constraint:*  $0 < \mathbf{iconst} < \mathbf{ip}$ .
  
- 3: **p[ip × ip + 2 × ip]** – const double *Input*  
*On entry:* **p** as returned by nag\_regsn\_mult\_linear (g02dac) and nag\_regsn\_mult\_linear\_upd\_model (g02ddc).
  
- 4: **c[ip × tdc]** – const double *Input*  
**Note:** the  $(i, j)$ th element of the matrix  $C$  is stored in  $\mathbf{c}[(i - 1) \times \mathbf{tdc} + j - 1]$ .  
*On entry:* the **iconst** constraints stored by column, i.e., the  $i$ th constraint is stored in the  $i$ th column of **c**.
  
- 5: **tdc** – Integer *Input*  
*On entry:* the stride separating matrix column elements in the array **c**.  
*Constraint:*  $\mathbf{tdc} \geq \mathbf{iconst}$ .
  
- 6: **b[ip]** – double *Input/Output*  
*On entry:* the parameter estimates computed by using the singular value decomposition,  $\hat{\beta}_{svd}$ .  
*On exit:* the parameter estimates of the arguments with the constraints imposed,  $\hat{\beta}_c$ .
  
- 7: **rss** – double *Input*  
*On entry:* the residual sum of squares as returned by nag\_regsn\_mult\_linear (g02dac) or nag\_regsn\_mult\_linear\_upd\_model (g02ddc).  
*Constraint:*  $\mathbf{rss} > 0.0$ .
  
- 8: **df** – double *Input*  
*On entry:* the degrees of freedom associated with the residual sum of squares as returned by nag\_regsn\_mult\_linear (g02dac) or nag\_regsn\_mult\_linear\_upd\_model (g02ddc).  
*Constraint:*  $\mathbf{df} > 0.0$ .
  
- 9: **se[ip]** – double *Output*  
*On exit:* the standard error of the parameter estimates in **b**.

10: **cov**[**ip** × (**ip** + 1)/2] – double *Output*

*On exit:* the upper triangular part of the variance-covariance matrix of the **ip** parameter estimates given in **b**. They are stored packed by column, i.e., the covariance between the parameter estimate given in **b**[*i*] and the parameter estimate given in **b**[*j*],  $j \geq i$ , is stored in **cov**[ $(j+1)/2 + i$ ], for  $i = 0, 1, \dots, \mathbf{ip} - 1$  and  $j = i, \dots, \mathbf{ip} - 1$ .

11: **fail** – NagError \* *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_2\_INT\_ARG\_GE

On entry, **iconst** =  $\langle value \rangle$  while **ip** =  $\langle value \rangle$ . These arguments must satisfy **iconst** < **ip**.

### NE\_2\_INT\_ARG\_LT

On entry, **tdc** =  $\langle value \rangle$  while **iconst** =  $\langle value \rangle$ . These arguments must satisfy **tdc** ≥ **iconst**.

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

### NE\_INT\_ARG\_LE

On entry, **iconst** =  $\langle value \rangle$ .  
Constraint: **iconst** > 0.

### NE\_INT\_ARG\_LT

On entry, **ip** =  $\langle value \rangle$ .  
Constraint: **ip** ≥ 1.

### NE\_MAT\_NOT\_FULL\_RANK

Matrix **c** does not give a model of full rank.

### NE\_REAL\_ARG\_LE

On entry, **df** must not be less than or equal to 0.0: **df** =  $\langle value \rangle$ .

On entry, **rss** must not be less than or equal to 0.0: **rss** =  $\langle value \rangle$ .

## 7 Accuracy

It should be noted that due to rounding errors an argument that should be zero when the constraints have been imposed may be returned as a value of order *machine precision*.

## 8 Parallelism and Performance

nag\_regn\_mult\_linear\_tran\_model (g02dkc) is not threaded in any implementation.

## 9 Further Comments

nag\_regn\_mult\_linear\_tran\_model (g02dkc) is intended for use in situations in which dummy (0-1) variables have been used such as in the analysis of designed experiments when you do not wish to change the arguments of the model to give a full rank model. The function is not intended for situations in which the relationships between the independent variables are only approximate.

## 10 Example

Data from an experiment with four treatments and three observations per treatment are read in. A model, including the mean term, is fitted by `nag_regsn_mult_linear` (g02dac) and the results printed. The constraint that the sum of treatment effects is zero is then read in and the parameter estimates with this constraint imposed are computed by `nag_regsn_mult_linear_tran_model` (g02dkc) and printed.

### 10.1 Program Text

```

/* nag_regsn_mult_linear_tran_model (g02dkc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg02.h>

#define X(I, J) x[(I) *tdx + J]
#define C(I, J) c[(I) *tdc + J]
int main(void)
{
    Integer exit_status = 0, i, iconst, ip, j, m, n, rank, *sx = 0, tdc,
        tdq, tdx;
    double df, rss, tol;
    double *b = 0, *c = 0, *com_ar = 0, *cov = 0, *h = 0, *p = 0;
    double *q = 0, *res = 0, *se = 0, *wt = 0, *wtptr, *x = 0, *y = 0;
    char nag_enum_arg[40];
    Nag_Boolean svd, weight;
    Nag_IncludeMean mean;
    NagError fail;

    INIT_FAIL(fail);

    printf("nag_regsn_mult_linear_tran_model (g02dkc) Example Program "
        "Results\n");
    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT " %" NAG_IFMT "", &n, &m);
#else
    scanf("%" NAG_IFMT " %" NAG_IFMT "", &n, &m);
#endif
#ifdef _WIN32
    scanf_s(" %39s", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf(" %39s", nag_enum_arg);
#endif
    /* nag_enum_name_to_value (x04nac).
     * Converts NAG enum member name to value
     */
    weight = (Nag_Boolean) nag_enum_name_to_value(nag_enum_arg);
#ifdef _WIN32
    scanf_s(" %39s", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf(" %39s", nag_enum_arg);
#endif
    mean = (Nag_IncludeMean) nag_enum_name_to_value(nag_enum_arg);
    if (n >= 2 && m >= 1) {
        if (!(h = NAG_ALLOC(n, double)) ||

```

```

        !(res = NAG_ALLOC(n, double)) ||
        !(wt = NAG_ALLOC(n, double)) ||
        !(x = NAG_ALLOC(n * m, double)) ||
        !(y = NAG_ALLOC(n, double)) || !(sx = NAG_ALLOC(m, Integer))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
    tdx = m;
}
else {
    printf("Invalid n.\n");
    exit_status = 1;
    return exit_status;
}
if (weight) {
    wtptr = wt;
    for (i = 0; i < n; i++) {
        for (j = 0; j < m; j++)
#ifdef _WIN32
            scanf_s("%lf", &X(i, j));
#else
            scanf("%lf", &X(i, j));
#endif
#ifdef _WIN32
            scanf_s("%lf%lf", &y[i], &wt[i]);
#else
            scanf("%lf%lf", &y[i], &wt[i]);
#endif
        }
    }
else {
    wtptr = (double *) 0;
    for (i = 0; i < n; i++) {
        for (j = 0; j < m; j++)
#ifdef _WIN32
            scanf_s("%lf", &X(i, j));
#else
            scanf("%lf", &X(i, j));
#endif
#ifdef _WIN32
            scanf_s("%lf", &y[i]);
#else
            scanf("%lf", &y[i]);
#endif
        }
    }
    for (j = 0; j < m; j++)
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "", &sx[j]);
#else
        scanf("%" NAG_IFMT "", &sx[j]);
#endif
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "", &ip);
#else
        scanf("%" NAG_IFMT "", &ip);
#endif

    if (!(b = NAG_ALLOC(ip, double)) ||
        !(c = NAG_ALLOC((ip) * (ip), double)) ||
        !(cov = NAG_ALLOC(ip * (ip + 1) / 2, double)) ||
        !(p = NAG_ALLOC(ip * (ip + 2), double)) ||
        !(q = NAG_ALLOC(n * (ip + 1), double)) ||
        !(se = NAG_ALLOC(ip, double)) ||
        !(com_ar = NAG_ALLOC(4 * ip * ip + 5 * (ip - 1), double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
}

```

```

}
tdq = ip + 1;
tdc = ip;

/* Set tolerance */
tol = 0.00001e0;
/* Find initial estimates using nag_regsn_mult_linear (g02dac) */
/* nag_regsn_mult_linear (g02dac).
 * Fits a general (multiple) linear regression model
 */
nag_regsn_mult_linear(mean, n, x, tdx, m, sx, ip, y, wtptr,
                    &rss, &df, b, se, cov, res, h, q, tdq,
                    &svd, &rank, p, tol, com_ar, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_regsn_mult_linear (g02dac).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

printf("Estimates from g02dac\n\n");
printf("Residual sum of squares = %13.4e\n", rss);
printf("Degrees of freedom = %3.1f\n\n", df);
printf("Variable   Parameter estimate   Standard error\n\n");
for (j = 0; j < ip; j++)
    printf("%6" NAG_IFMT "%20.4e%20.4e\n", j + 1, b[j], se[j]);
printf("\n");
/*
 *   Input constraints and call nag_regsn_mult_linear_tran_model (g02dkc)
 */
iconst = ip - rank;
for (i = 0; i < ip; ++i)
    for (j = 0; j < iconst; ++j)
#ifdef _WIN32
        scanf_s("%lf", &C(i, j));
#else
        scanf("%lf", &C(i, j));
#endif

/* nag_regsn_mult_linear_tran_model (g02dkc).
 * Estimates of parameters of a general linear regression
 * model for given constraints
 */
nag_regsn_mult_linear_tran_model(ip, iconst, p, c, tdc, b, rss, df, se, cov,
                                &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_regsn_mult_linear_tran_model (g02dkc).\n%s\n",
          fail.message);
    exit_status = 1;
    goto END;
}

printf("\n");
printf("Estimates from nag_regsn_mult_linear_tran_model (g02dkc) using "
      "constraints\n\n");
printf("Variable   Parameter estimate   Standard error\n\n");
for (j = 0; j < ip; j++)
    printf("%6" NAG_IFMT "%20.4e%20.4e\n", j + 1, b[j], se[j]);
printf("\n");
END:
NAG_FREE(h);
NAG_FREE(res);
NAG_FREE(wt);
NAG_FREE(x);
NAG_FREE(y);
NAG_FREE(sx);
NAG_FREE(b);
NAG_FREE(c);
NAG_FREE(cov);
NAG_FREE(p);
NAG_FREE(q);

```

```

NAG_FREE(se);
NAG_FREE(com_ar);

return exit_status;
}

```

## 10.2 Program Data

```

nag_regsn_mult_linear_tran_model (g02dkc) Example Program Data
 12 4 Nag_FALSE Nag_MeanInclude
1.0 0.0 0.0 0.0 33.63
0.0 0.0 0.0 1.0 39.62
0.0 1.0 0.0 0.0 38.18
0.0 0.0 1.0 0.0 41.46
0.0 0.0 0.0 1.0 38.02
0.0 1.0 0.0 0.0 35.83
0.0 0.0 0.0 1.0 35.99
1.0 0.0 0.0 0.0 36.58
0.0 0.0 1.0 0.0 42.92
1.0 0.0 0.0 0.0 37.80
0.0 0.0 1.0 0.0 40.43
0.0 1.0 0.0 0.0 37.89
 1 1 1 1 5
0.0
1.0
1.0
1.0
1.0

```

## 10.3 Program Results

```

nag_regsn_mult_linear_tran_model (g02dkc) Example Program Results
Estimates from g02dac

```

```

Residual sum of squares = 2.2227e+01
Degrees of freedom = 8.0

```

Variable	Parameter estimate	Standard error
1	3.0557e+01	3.8494e-01
2	5.4467e+00	8.3896e-01
3	6.7433e+00	8.3896e-01
4	1.1047e+01	8.3896e-01
5	7.3200e+00	8.3896e-01

```

Estimates from nag_regsn_mult_linear_tran_model (g02dkc) using constraints

```

Variable	Parameter estimate	Standard error
1	3.8196e+01	4.8117e-01
2	-2.1925e+00	8.3342e-01
3	-8.9583e-01	8.3342e-01
4	3.4075e+00	8.3342e-01
5	-3.1917e-01	8.3342e-01

---