NAG Library Function Document

nag_sparse_herm_precon_ichol_solve (f11jpc)

1 Purpose

nag_sparse_herm_precon_ichol_solve (f11jpc) solves a system of complex linear equations involving the incomplete Cholesky preconditioning matrix generated by nag_sparse_herm_chol_fac (f11jnc).

2 Specification

```
#include <nag.h>
#include <nagf11.h>
```

```
void nag_sparse_herm_precon_ichol_solve (Integer n, const Complex a[],
     Integer la, const Integer irow[], const Integer icol[],
     const Integer ipiv[], const Integer istr[],
     Nag_SparseSym_CheckData check, const Complex y[], Complex x[],
     NagError *fail)
```

3 Description

nag_sparse_herm_precon_ichol_solve (f11jpc) solves a system of linear equations

$$Mx = y$$

involving the preconditioning matrix $M = PLDL^{\mathrm{H}}P^{\mathrm{T}}$, corresponding to an incomplete Cholesky decomposition of a complex sparse Hermitian matrix stored in symmetric coordinate storage (SCS) format (see Section 2.1.2 in the f11 Chapter Introduction), as generated by nag_sparse_herm_chol_fac (f11jnc).

In the above decomposition L is a complex lower triangular sparse matrix with unit diagonal, D is a real diagonal matrix and P is a permutation matrix. L and D are supplied to nag_sparse_herm_precon_ichol_solve (f11jpc) through the matrix

$$C = L + D^{-1} - I$$

which is a lower triangular n by n complex sparse matrix, stored in SCS format, as returned by nag_sparse_herm_chol_fac (f11jnc). The permutation matrix P is returned from nag_sparse_herm_chol_fac (f11jnc) via the array **ipiv**.

nag_sparse_herm_precon_ichol_solve (f11jpc) may also be used in combination with nag_sparse_herm_chol_fac (f11jnc) to solve a sparse complex Hermitian positive definite system of linear equations directly (see nag_sparse_herm_chol_fac (f11jnc)). This is illustrated in Section 10.

4 References

None.

5 Arguments

1: **n** – Integer *Input*

On entry: n, the order of the matrix M. This **must** be the same value as was supplied in the preceding call to nag_sparse_herm_chol_fac (f11jnc).

Constraint: $\mathbf{n} \geq 1$.

2: **a**[**la**] – const Complex *Input*

On entry: the values returned in the array **a** by a previous call to nag_sparse_herm_chol_fac (f11jnc).

3: **la** – Integer *Input*

On entry: the dimension of the arrays **a**, **irow** and **icol**. This **must** be the same value supplied in the preceding call to nag_sparse_herm_chol_fac (f11jnc).

4: **irow**[**la**] – const Integer *Input*
5: **icol**[**la**] – const Integer *Input*
6: **ipiv**[**n**] – const Integer *Input*
7: **istr**[$\mathbf{n}+\mathbf{1}$] – const Integer *Input*

On entry: the values returned in arrays **irow**, **icol**, **ipiv** and **istr** by a previous call to nag_sparse_herm_chol_fac (f11jnc).

8: **check** – Nag_SparseSym_CheckData *Input*

On entry: specifies whether or not the input data should be checked.

check = Nag_SparseSym_Check
Checks are carried out on the values of **n**, **irow**, **icol**, **ipiv** and **istr**.

check = Nag_SparseSym_NoCheck
None of these checks are carried out.

Constraint: **check** = Nag_SparseSym_Check or Nag_SparseSym_NoCheck.

9: **y**[**n**] – const Complex *Input*

On entry: the right-hand side vector y.

10: **x**[**n**] – Complex *Output*

On exit: the solution vector x.

11: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: $\mathbf{n} \geq 1$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in How to Use the NAG Library and its Documentation for further information.

NE_INVALID_ROWCOL_PIVOT

Check that **a**, **irow**, **icol**, **ipiv** and **istr** have not been corrupted between calls to nag_sparse_herm_chol_fac (f11jnc) and nag_sparse_herm_precon_ichol_solve (f11jpc).

NE_INVALID_SCS

Check that **a**, **irow**, **icol**, **ipiv** and **istr** have not been corrupted between calls to nag_sparse_herm_chol_fac (f11jnc) and nag_sparse_herm_precon_ichol_solve (f11jpc).

NE_INVALID_SCS_PRECOND

Check that **a**, **irow**, **icol**, **ipiv** and **istr** have not been corrupted between calls to nag_sparse_herm_chol_fac (f11jnc) and nag_sparse_herm_precon_ichol_solve (f11jpc).

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in How to Use the NAG Library and its Documentation for further information.

NE_NOT_STRICTLY_INCREASING

Check that **a**, **irow**, **icol**, **ipiv** and **istr** have not been corrupted between calls to nag_sparse_herm_chol_fac (f11jnc) and nag_sparse_herm_precon_ichol_solve (f11jpc).

7 Accuracy

The computed solution x is the exact solution of a perturbed system of equations $(M + \delta M)x = y$, where

$$|\delta M| \leq c(n)\epsilon P|L||D|\left|L^{\mathrm{H}}\right|P^{\mathrm{T}},$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

8 Parallelism and Performance

nag_sparse_herm_precon_ichol_solve (f11jpc) is not threaded in any implementation.

9 Further Comments

9.1 Timing

The time taken for a call to nag_sparse_herm_precon_ichol_solve (f11jpc) is proportional to the value of **nnzc** returned from nag_sparse_herm_chol_fac (f11jnc).

10 Example

This example reads in a complex sparse Hermitian positive definite matrix A and a vector y. It then calls nag_sparse_herm_chol_fac (f11jnc), with **lfill** $= -1$ and **dtol** $= 0.0$, to compute the **complete** Cholesky decomposition of A:

$$A = PLDL^{\mathrm{H}}P^{\mathrm{T}}.$$

Finally it calls nag_sparse_herm_precon_ichol_solve (f11jpc) to solve the system

$$PLDL^{\mathrm{H}}P^{\mathrm{T}}x = y.$$

10.1 Program Text

```
/* nag_sparse_herm_precon_ichol_solve (f11jpc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <nag_stdlib.h>
#include <naga02.h>
#include <nagf11.h>

int main(void)
{
  /* Scalars */
  Integer exit_status = 0;
  double dscale, dtol;
  Integer i, la, lfill, n, nnz, nnzc, npivm;
  /* Arrays */
  Complex *a = 0, *x = 0, *y = 0;
  Integer *icol = 0, *ipiv = 0, *irow = 0, *istr = 0;
  /* NAG types */
  Nag_SparseSym_Fact mic;
  Nag_SparseSym_Piv pstrat;
  Nag_SparseSym_CheckData check;
  NagError fail;

  INIT_FAIL(fail);

  printf("nag_sparse_herm_precon_ichol_solve (f11jpc) Example Program Results");
  printf("\n\n");
  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif
#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%*[^\n]", &n);
#else
  scanf("%" NAG_IFMT "%*[^\n]", &n);
#endif
#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%*[^\n]", &nnz);
#else
  scanf("%" NAG_IFMT "%*[^\n]", &nnz);
#endif

  /* Allocate memory */
  la = 3 * nnz;
  if (!(a = NAG_ALLOC(la, Complex)) ||
      !(x = NAG_ALLOC(n, Complex)) ||
      !(y = NAG_ALLOC(n, Complex)) ||
      !(icol = NAG_ALLOC(la, Integer)) ||
      !(ipiv = NAG_ALLOC(n, Integer)) ||
      !(irow = NAG_ALLOC(la, Integer)) || !(istr = NAG_ALLOC(n + 1, Integer))
         )
  {
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
  }
  /* Read the matrix a */
  for (i = 0; i <= nnz - 1; i++)
#ifdef _WIN32
    scanf_s(" ( %lf , %lf ) %" NAG_IFMT "%" NAG_IFMT "%*[^\n] ",
            &a[i].re, &a[i].im, &irow[i], &icol[i]);
```

```
#else
    scanf(" ( %lf , %lf ) %" NAG_IFMT "%" NAG_IFMT "%*[^\n] ",
          &a[i].re, &a[i].im, &irow[i], &icol[i]);
#endif
  /* Read the vector y */
  for (i = 0; i <= n - 1; i++)
#ifdef _WIN32
    scanf_s(" ( %lf , %lf ) ", &y[i].re, &y[i].im);
#else
    scanf(" ( %lf , %lf ) ", &y[i].re, &y[i].im);
#endif

  lfill = -1;
  dtol = 0.0;
  dscale = 0.0;
  mic = Nag_SparseSym_UnModFact;
  pstrat = Nag_SparseSym_MarkPiv;
  /* Calculate Cholesky factorization using  nag_sparse_herm_chol_fac (f11jnc).
   */
  nag_sparse_herm_chol_fac(n, nnz, a, la, irow, icol, lfill, dtol, mic,
                           dscale, pstrat, ipiv, istr, &nnzc, &npivm, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_sparse_herm_chol_fac (f11jnc).\n%s\n",
           fail.message);
    exit_status = 1;
    goto END;
  }
  /* Check the output value of npivm */
  if (npivm != 0)
    printf("Factorization is not complete \n");
  else {
    /* Solve complex linear system involving incomplete Cholesky factorization
     *
     *             H T
     *       P L D L P x = y
     *
     * using nag_sparse_herm_precon_ichol_solve (f11jpc).
     */
    check = Nag_SparseSym_Check;
    nag_sparse_herm_precon_ichol_solve(n, a, la, irow, icol, ipiv, istr,
                                       check, y, x, &fail);
    if (fail.code != NE_NOERROR) {
      printf("Error from nag_sparse_herm_precon_ichol_solve (f11jpc).\n%s\n",
             fail.message);
      exit_status = 2;
      goto END;
    }
    /* Output results */
    printf("Solution of linear system \n");
    for (i = 0; i <= n - 1; i++)
      printf(" (%13.4e, %13.4e) \n", x[i].re, x[i].im);
  }

END:
  NAG_FREE(a);
  NAG_FREE(x);
  NAG_FREE(y);
  NAG_FREE(icol);
  NAG_FREE(ipiv);
  NAG_FREE(irow);
  NAG_FREE(istr);
  return exit_status;
}
```

10.2 Program Data

```
nag_sparse_herm_precon_ichol_solve (f11jpc) Example Program Data
  9                    : n
 23                    : nnz
 ( 6., 0.)   1    1
 (-1., 1.)   2    1
 ( 6., 0.)   2    2
 ( 0., 1.)   3    2
 ( 5., 0.)   3    3
 ( 5., 0.)   4    4
 ( 2.,-2.)   5    1
 ( 4., 0.)   5    5
 ( 1., 1.)   6    3
 ( 2., 0.)   6    4
 ( 6., 0.)   6    6
 (-4., 3.)   7    2
 ( 0., 1.)   7    5
 (-1., 0.)   7    6
 ( 6., 0.)   7    7
 (-1.,-1.)   8    4
 ( 0.,-1.)   8    6
 ( 9., 0.)   8    8
 ( 1., 3.)   9    1
 ( 1., 2.)   9    5
 (-1., 0.)   9    6
 ( 1., 4.)   9    8
 ( 9., 0.)   9    9     : a[i], irow[i], icol[i], i=0,...,nnz-1
 ( 8.,54.) (-10.,-92.)
 (25.,27.) (26., -28.)
 (54.,12.) (26.,-22.)
 (47.,65.) (71.,-57.)
 (60.,70.)              : y[i], i=0,...,n-1
```

10.3 Program Results

```
nag_sparse_herm_precon_ichol_solve (f11jpc) Example Program Results

Solution of linear system
 (   1.0000e+00,    9.0000e+00)
 (   2.0000e+00,   -8.0000e+00)
 (   3.0000e+00,    7.0000e+00)
 (   4.0000e+00,   -6.0000e+00)
 (   5.0000e+00,    5.0000e+00)
 (   6.0000e+00,   -4.0000e+00)
 (   7.0000e+00,    3.0000e+00)
 (   8.0000e+00,   -2.0000e+00)
 (   9.0000e+00,    1.0000e+00)
```