

## NAG Library Function Document

### nag\_sparse\_nherm\_precon\_bdilu\_solve (f11duc)

#### 1 Purpose

nag\_sparse\_nherm\_precon\_bdilu\_solve (f11duc) solves a complex sparse non-Hermitian system of linear equations, represented in coordinate storage format, using a restarted generalized minimal residual (RGMRES), conjugate gradient squared (CGS), stabilized bi-conjugate gradient (Bi-CGSTAB), or transpose-free quasi-minimal residual (TFQMR) method, with block Jacobi or additive Schwarz preconditioning.

#### 2 Specification

```
#include <nag.h>
#include <nagf11.h>

void nag_sparse_nherm_precon_bdilu_solve (Nag_SparseNsym_Method method,
    Integer n, Integer nnz, const Complex a[], Integer la,
    const Integer irow[], const Integer icol[], Integer nb,
    const Integer istb[], const Integer indb[], Integer lindb,
    const Integer ipivp[], const Integer ipivq[], const Integer istr[],
    const Integer idiag[], const Complex b[], Integer m, double tol,
    Integer maxitn, Complex x[], double *rnorm, Integer *itn,
    NagError *fail)
```

#### 3 Description

nag\_sparse\_nherm\_precon\_bdilu\_solve (f11duc) solves a complex sparse non-Hermitian linear system of equations

$$Ax = b,$$

using a preconditioned RGMRES (see Saad and Schultz (1986)), CGS (see Sonneveld (1989)), Bi-CGSTAB( $\ell$ ) (see Van der Vorst (1989) and Sleijpen and Fokkema (1993)), or TFQMR (see Freund and Nachtigal (1991) and Freund (1993)) method.

nag\_sparse\_nherm\_precon\_bdilu\_solve (f11duc) uses the incomplete (possibly overlapping) block  $LU$  factorization determined by nag\_sparse\_nherm\_precon\_bdilu (f11dtc) as the preconditioning matrix. A call to nag\_sparse\_nherm\_precon\_bdilu\_solve (f11duc) must always be preceded by a call to nag\_sparse\_nherm\_precon\_bdilu (f11dtc). Alternative preconditioners for the same storage scheme are available by calling nag\_sparse\_nherm\_fac\_sol (f11dq) or nag\_sparse\_nherm\_sol (f11dsc).

The matrix  $A$ , and the preconditioning matrix  $M$ , are represented in coordinate storage (CS) format (see Section 2.1.1 in the f11 Chapter Introduction) in the arrays **a**, **irow** and **icol**, as returned from nag\_sparse\_nherm\_precon\_bdilu (f11dtc). The array **a** holds the nonzero entries in these matrices, while **irow** and **icol** hold the corresponding row and column indices.

nag\_sparse\_nherm\_precon\_bdilu\_solve (f11duc) is a Black Box function which calls nag\_sparse\_nherm\_basic\_setup (f11brc), nag\_sparse\_nherm\_basic\_solver (f11bsc) and nag\_sparse\_nherm\_basic\_diagnostic (f11btc). If you wish to use an alternative storage scheme, preconditioner, or termination criterion, or require additional diagnostic information, you should call these underlying functions directly.

## 4 References

Freund R W (1993) A transpose-free quasi-minimal residual algorithm for non-Hermitian linear systems *SIAM J. Sci. Comput.* **14** 470–482

Freund R W and Nachtigal N (1991) QMR: a Quasi-Minimal Residual Method for Non-Hermitian Linear Systems *Numer. Math.* **60** 315–339

Saad Y and Schultz M (1986) GMRES: a generalized minimal residual algorithm for solving nonsymmetric linear systems *SIAM J. Sci. Statist. Comput.* **7** 856–869

Sleijpen G L G and Fokkema D R (1993) BiCGSTAB( $\ell$ ) for linear equations involving matrices with complex spectrum *ETNA* **1** 11–32

Sonneveld P (1989) CGS, a fast Lanczos-type solver for nonsymmetric linear systems *SIAM J. Sci. Statist. Comput.* **10** 36–52

Van der Vorst H (1989) Bi-CGSTAB, a fast and smoothly converging variant of Bi-CG for the solution of nonsymmetric linear systems *SIAM J. Sci. Statist. Comput.* **13** 631–644

## 5 Arguments

1: **method** – Nag\_SparseNsym\_Method *Input*

*On entry:* specifies the iterative method to be used.

**method** = Nag\_SparseNsym\_RGMRES  
Restarted generalized minimum residual method.

**method** = Nag\_SparseNsym\_CGS  
Conjugate gradient squared method.

**method** = Nag\_SparseNsym\_BiCGSTAB  
Bi-conjugate gradient stabilized ( $\ell$ ) method.

**method** = Nag\_SparseNsym\_TFQMR  
Transpose-free quasi-minimal residual method.

*C o n s t r a i n t :* **method** = Nag\_SparseNsym\_RGMRES, Nag\_SparseNsym\_CGS,  
Nag\_SparseNsym\_BiCGSTAB or Nag\_SparseNsym\_TFQMR.

2: **n** – Integer *Input*

3: **nnz** – Integer *Input*

4: **a[la]** – const Complex *Input*

5: **la** – Integer *Input*

6: **irow[la]** – const Integer *Input*

7: **icol[la]** – const Integer *Input*

8: **nb** – Integer *Input*

9: **istb[nb + 1]** – const Integer *Input*

10: **indb[lindb]** – const Integer *Input*

11: **lindb** – Integer *Input*

12: **ipivp[lindb]** – const Integer *Input*

13: **ipivq[lindb]** – const Integer *Input*

14: **istr[lindb + 1]** – const Integer *Input*

15: **idiag[lindb]** – const Integer *Input*

*On entry:* the values returned in arrays **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** by a previous call to nag\_sparse\_nherm\_precon\_bdilu (f11dtc).

The arrays **istb**, **indb** and **a** together with the scalars **n**, **nnz**, **la**, **nb** and **lindb** must be the same values that were supplied in the preceding call to nag\_sparse\_nherm\_precon\_bdilu (f11dtc).

*On entry:* the values returned in arrays **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** by a previous call to nag\_sparse\_nherm\_precon\_bdilu (f11dtc).

The arrays **istb**, **indb** and the scalars **nb** and **lindb** must be the same values that were supplied in the preceding call to `nag_sparse_nherm_precon_bdilu` (f11dtc).

16: **b[n]** – const Complex *Input*

*On entry:* the right-hand side vector  $b$ .

17: **m** – Integer *Input*

*On entry:* if **method** = Nag\_SparseNsym\_RGMRES, **m** is the dimension of the restart subspace.

If **method** = Nag\_SparseNsym\_BiCGSTAB, **m** is the order  $\ell$  of the polynomial Bi-CGSTAB method.

Otherwise, **m** is not referenced.

*Constraints:*

if **method** = Nag\_SparseNsym\_RGMRES,  $0 < \mathbf{m} \leq \min(\mathbf{n}, 50)$ ;

if **method** = Nag\_SparseNsym\_BiCGSTAB,  $0 < \mathbf{m} \leq \min(\mathbf{n}, 10)$ .

18: **tol** – double *Input*

*On entry:* the required tolerance. Let  $x_k$  denote the approximate solution at iteration  $k$ , and  $r_k$  the corresponding residual. The algorithm is considered to have converged at iteration  $k$  if

$$\|r_k\|_\infty \leq \tau \times (\|b\|_\infty + \|A\|_\infty \|x_k\|_\infty).$$

If **tol**  $\leq 0.0$ ,  $\tau = \max(\sqrt{\epsilon}, \sqrt{n}\epsilon)$  is used, where  $\epsilon$  is the *machine precision*. Otherwise  $\tau = \max(\mathbf{tol}, 10\epsilon, \sqrt{n}\epsilon)$  is used.

*Constraint:* **tol**  $< 1.0$ .

19: **maxitn** – Integer *Input*

*On entry:* the maximum number of iterations allowed.

*Constraint:* **maxitn**  $\geq 1$ .

20: **x[n]** – Complex *Input/Output*

*On entry:* an initial approximation to the solution vector  $x$ .

*On exit:* an improved approximation to the solution vector  $x$ .

21: **rnorm** – double \* *Output*

*On exit:* the final value of the residual norm  $\|r_k\|_\infty$ , where  $k$  is the output value of **itn**.

22: **itn** – Integer \* *Output*

*On exit:* the number of iterations carried out.

23: **fail** – NagError \* *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ACCURACY

The required accuracy could not be obtained. However, a reasonable accuracy may have been achieved. You should check the output value of **rnorm** for acceptability. This error code usually implies that your problem has been fully and satisfactorily solved to within or close to the accuracy available on your system. Further iterations are unlikely to improve on this situation.

**NE\_ALG\_FAIL**

Algorithmic breakdown. A solution is returned, although it is possible that it is completely inaccurate.

**NE\_ALLOC\_FAIL**

Dynamic memory allocation failed.

See Section 3.2.1.2 in How to Use the NAG Library and its Documentation for further information.

**NE\_BAD\_PARAM**

On entry, argument  $\langle value \rangle$  had an illegal value.

**NE\_CONVERGENCE**

The solution has not converged after  $\langle value \rangle$  iterations.

**NE\_INT**

On entry,  $\mathbf{istb}[0] = \langle value \rangle$ .

Constraint:  $\mathbf{istb}[0] \geq 1$ .

On entry,  $\mathbf{maxitn} = \langle value \rangle$ .

Constraint:  $\mathbf{maxitn} \geq 1$ .

On entry,  $\mathbf{n} = \langle value \rangle$ .

Constraint:  $\mathbf{n} \geq 1$ .

On entry,  $\mathbf{nnz} = \langle value \rangle$ .

Constraint:  $\mathbf{nnz} \geq 1$ .

**NE\_INT\_2**

On entry,  $\mathbf{la} = \langle value \rangle$  and  $\mathbf{nnz} = \langle value \rangle$ .

Constraint:  $\mathbf{la} \geq 2 \times \mathbf{nnz}$ .

On entry,  $\mathbf{nb} = \langle value \rangle$  and  $\mathbf{n} = \langle value \rangle$ .

Constraint:  $1 \leq \mathbf{nb} \leq \mathbf{n}$ .

On entry,  $\mathbf{nnz} = \langle value \rangle$  and  $\mathbf{n} = \langle value \rangle$ .

Constraint:  $\mathbf{nnz} \leq \mathbf{n}^2$ .

**NE\_INT\_3**

On entry,  $\mathbf{lindb} = \langle value \rangle$ ,  $\mathbf{istb}[\mathbf{nb}] - 1 = \langle value \rangle$  and  $\mathbf{nb} = \langle value \rangle$ .

Constraint:  $\mathbf{lindb} \geq \mathbf{istb}[\mathbf{nb}] - 1$ .

On entry,  $\mathbf{m} = \langle value \rangle$  and  $\mathbf{n} = \langle value \rangle$ .

Constraint: if  $\mathbf{method} = \text{Nag\_SparseNsym\_RGMRES}$ ,  $1 \leq \mathbf{m} \leq \min(\mathbf{n}, \langle value \rangle)$ .

If  $\mathbf{method} = \text{Nag\_SparseNsym\_BiCGSTAB}$ ,  $1 \leq \mathbf{m} \leq \min(\mathbf{n}, \langle value \rangle)$ .

**NE\_INT\_ARRAY**

On entry,  $\mathbf{indb}[\langle value \rangle] = \langle value \rangle$  and  $\mathbf{n} = \langle value \rangle$ .

Constraint:  $1 \leq \mathbf{indb}[m - 1] \leq \mathbf{n}$ , for  $m = \mathbf{istb}[0], \mathbf{istb}[0] + 1, \dots, \mathbf{istb}[\mathbf{nb}] - 1$ .

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 3.6.6 in How to Use the NAG Library and its Documentation for further information.

**NE\_INVALID\_CS**

On entry, **icol**[*value*] = *value* and **n** = *value*.

Constraint:  $1 \leq \mathbf{icol}[i-1] \leq \mathbf{n}$ , for  $i = 1, 2, \dots, \mathbf{nnz}$ .

Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to `nag_sparse_nherm_precon_bdilu` (f11dtc) and `nag_sparse_nherm_precon_bdilu_solve` (f11duc).

On entry, **irow**[*value*] = *value* and **n** = *value*.

Constraint:  $1 \leq \mathbf{irow}[i-1] \leq \mathbf{n}$ , for  $i = 1, 2, \dots, \mathbf{nnz}$ .

Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to `nag_sparse_nherm_precon_bdilu` (f11dtc) and `nag_sparse_nherm_precon_bdilu_solve` (f11duc).

**NE\_INVALID\_CS\_PRECOND**

The CS representation of the preconditioner is invalid.

Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to `nag_sparse_nherm_precon_bdilu` (f11dtc) and `nag_sparse_nherm_precon_bdilu_solve` (f11duc).

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.

See Section 3.6.5 in How to Use the NAG Library and its Documentation for further information.

**NE\_NOT\_STRICTLY\_INCREASING**

On entry, element *value* of **a** was out of order.

Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to `nag_sparse_nherm_precon_bdilu` (f11dtc) and `nag_sparse_nherm_precon_bdilu_solve` (f11duc).

On entry, for  $b = \langle \text{value} \rangle$ , **istb**[ $b$ ] = *value* and **istb**[ $b-1$ ] = *value*.

Constraint:  $\mathbf{istb}[b] > \mathbf{istb}[b-1]$ , for  $b = 1, 2, \dots, \mathbf{nb}$ .

On entry, location *value* of (**irow**, **icol**) was a duplicate.

Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to `nag_sparse_nherm_precon_bdilu` (f11dtc) and `nag_sparse_nherm_precon_bdilu_solve` (f11duc).

**NE\_REAL**

On entry, **tol** = *value*.

Constraint: **tol** < 1.0.

**7 Accuracy**

On successful termination, the final residual  $r_k = b - Ax_k$ , where  $k = \mathbf{itn}$ , satisfies the termination criterion

$$\|r_k\|_\infty \leq \tau \times (\|b\|_\infty + \|A\|_\infty \|x_k\|_\infty).$$

The value of the final residual norm is returned in **rnorm**.

**8 Parallelism and Performance**

`nag_sparse_nherm_precon_bdilu_solve` (f11duc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

`nag_sparse_nherm_precon_bdilu_solve` (f11duc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The time taken by `nag_sparse_nherm_precon_bdilu_solve` (f11duc) for each iteration is roughly proportional to the value of `nnzc` returned from the preceding call to `nag_sparse_nherm_precon_bdilu` (f11dtc).

The number of iterations required to achieve a prescribed accuracy cannot be easily determined *a priori*, as it can depend dramatically on the conditioning and spectrum of the preconditioned coefficient matrix  $\bar{A} = M^{-1}A$ .

## 10 Example

This example program reads in a sparse matrix  $A$  and a vector  $b$ . It calls `nag_sparse_nherm_precon_bdilu` (f11dtc), with the array `lfill = 0` and the array `dtol = 0.0`, to compute an overlapping incomplete  $LU$  factorization. This is then used as an additive Schwarz preconditioner on a call to `nag_sparse_nherm_precon_bdilu_solve` (f11duc) which uses the RGMRES method to solve  $Ax = b$ .

### 10.1 Program Text

```
/* nag_sparse_nherm_precon_bdilu_solve (f11duc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <nagf11.h>
#include <nag_stdlib.h>

static void overlap(Integer *n, Integer *nnz, Integer *irow, Integer *icol,
                   Integer *nb, Integer *istb, Integer *indb, Integer *lindb,
                   Integer *nover, Integer *iwork);

int main(void)
{
    /* Scalars */
    double dtolg, rnorm, tol;
    Integer i, itn, k, la, lfillg, lindb, liwork, m, maxitn, mb, n, nb, nnz;
    Integer nnzc, nover, exit_status = 0;
    Nag_SparseNsym_Method method;
    Nag_SparseNsym_Piv pstrag;
    Nag_SparseNsym_Fact milug;

    /* Arrays */
    char nag_enum_arg[40];
    double *dtol;
    Complex *a, *b, *x;
    Integer *icol, *idiag, *indb, *ipivp, *ipivq, *irow, *istb, *istr, *iwork;
    Integer *lfill, *npivm;
    Nag_SparseNsym_Piv *pstrat;
    Nag_SparseNsym_Fact *milu;

    /* Nag Types */
    NagError fail;

    /* Print example header */
    printf("nag_sparse_nherm_precon_bdilu_solve (f11duc) Example Program ");
    printf("Results\n\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else

```

```

    scanf("%*[\n] ");
#endif

/* Get the matrix order and number of nonzero entries. */
#ifdef _WIN32
    scanf_s("%" NAG_IFMT " %*[\n]", &n);
#else
    scanf("%" NAG_IFMT " %*[\n]", &n);
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT " %*[\n]", &nnz);
#else
    scanf("%" NAG_IFMT " %*[\n]", &nnz);
#endif

    la = 20 * nnz;
    lindb = 3 * n;
    liwork = 9 * n + 3;

/* Allocate arrays */
    b = NAG_ALLOC(n, Complex);
    x = NAG_ALLOC(n, Complex);

    a = NAG_ALLOC(la, Complex);
    irow = NAG_ALLOC(la, Integer);
    icol = NAG_ALLOC(la, Integer);

    iddiag = NAG_ALLOC(lindb, Integer);
    indb = NAG_ALLOC(lindb, Integer);
    ipivp = NAG_ALLOC(lindb, Integer);
    ipivq = NAG_ALLOC(lindb, Integer);
    istr = NAG_ALLOC(lindb + 1, Integer);

    iwork = NAG_ALLOC(liwork, Integer);

    if ((!b) || (!x) || (!a) || (!irow) || (!icol) || (!iddiag) || (!indb) ||
        (!ipivp) || (!ipivq) || (!istr) || (!iwork)) {
        printf("Allocation failure!\n");
        exit_status = -1;
        goto END;
    }

/* Initialize arrays */
    for (i = 0; i < n; i++) {
        b[i].re = 0.0;
        b[i].im = 0.0;
        x[i].re = 0.0;
        x[i].im = 0.0;
    }

    for (i = 0; i < la; i++) {
        a[i].re = 0.0;
        a[i].im = 0.0;
        irow[i] = 0;
        icol[i] = 0;
    }

    for (i = 0; i < lindb; i++) {
        indb[i] = 0;
        ipivp[i] = 0;
        ipivq[i] = 0;
        istr[i] = 0;
        iddiag[i] = 0;
    }
    istr[lindb] = 0;

    for (i = 0; i < liwork; i++) {
        iwork[i] = 0;
    }

/* Read the matrix A */

```

```

    for (i = 0; i < nnz; i++) {
#ifdef _WIN32
        scanf_s(" (%lf, %lf) %" NAG_IFMT " %" NAG_IFMT "",
                &a[i].re, &a[i].im, &irow[i], &icol[i]);
#else
        scanf(" (%lf, %lf) %" NAG_IFMT " %" NAG_IFMT "",
                &a[i].re, &a[i].im, &irow[i], &icol[i]);
#endif
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    /* Read the RHS b */
    for (i = 0; i < n; i++) {
#ifdef _WIN32
        scanf_s(" (%lf, %lf)", &b[i].re, &b[i].im);
#else
        scanf(" (%lf, %lf)", &b[i].re, &b[i].im);
#endif
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    /* nag_enum_name_to_value (x04nac): Converts NAG enum member name to value */
#ifdef _WIN32
    scanf_s("%39s %*[\n]", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf("%39s %*[\n]", nag_enum_arg);
#endif
    method = (Nag_SparseNsym_Method) nag_enum_name_to_value(nag_enum_arg);

    /* Read algorithmic parameters */
#ifdef _WIN32
    scanf_s("%" NAG_IFMT " %lf %*[\n]", &lfillg, &dtolg);
#else
    scanf("%" NAG_IFMT " %lf %*[\n]", &lfillg, &dtolg);
#endif

    /* nag_enum_name_to_value (x04nac): Converts NAG enum member name to value */
#ifdef _WIN32
    scanf_s("%39s %*[\n]", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf("%39s %*[\n]", nag_enum_arg);
#endif
    pstrag = (Nag_SparseNsym_Piv) nag_enum_name_to_value(nag_enum_arg);

    /* nag_enum_name_to_value (x04nac): Converts NAG enum member name to value */
#ifdef _WIN32
    scanf_s("%39s %*[\n]", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf("%39s %*[\n]", nag_enum_arg);
#endif
    milug = (Nag_SparseNsym_Fact) nag_enum_name_to_value(nag_enum_arg);

    /* Read algorithmic parameters */
#ifdef _WIN32
    scanf_s("%" NAG_IFMT " %lf %" NAG_IFMT " %*[\n]", &m, &tol, &maxitn);
#else
    scanf("%" NAG_IFMT " %lf %" NAG_IFMT " %*[\n]", &m, &tol, &maxitn);
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT " %" NAG_IFMT " %*[\n]", &nb, &nover);
#else
    scanf("%" NAG_IFMT " %" NAG_IFMT " %*[\n]", &nb, &nover);
#endif

```



```

if (nb < 1)
{
    printf("Value read for nb is out of range\n");
    exit_status = -4;
    goto END;
}

/* Allocate arrays */
dtol = NAG_ALLOC(nb, double);
istb = NAG_ALLOC(nb + 1, Integer);
lfill = NAG_ALLOC(nb, Integer);
npivm = NAG_ALLOC(nb, Integer);

pstrat = (Nag_SparseNsym_Piv *) NAG_ALLOC(nb, Nag_SparseNsym_Piv);
milu = (Nag_SparseNsym_Fact *) NAG_ALLOC(nb, Nag_SparseNsym_Fact);

if ((!dtol) || (!istb) || (!lfill) || (!npivm) || (!pstrat) || (!milu)) {
    printf("Allocation failure!\n");
    exit_status = -1;
    goto END;
}

/* Initialize arrays */
for (i = 0; i < nb; i++) {
    dtol[i] = 0.0;
    istb[i] = 0;
    lfill[i] = 0;
    npivm[i] = 0;
}
istb[nb] = 0;

/* Define diagonal block indices.
 * In this example use blocks of MB consecutive rows and initialize
 * assuming no overlap.
 */
mb = (n + nb - 1) / nb;
for (k = 0; k < nb; k++) {
    istb[k] = k * mb + 1;
}
istb[nb] = n + 1;

for (i = 0; i < n; i++) {
    indb[i] = i + 1;
}

/* Modify INDB and ISTB to account for overlap. */
overlap(&n, &nnz, irow, icol, &nb, istb, indb, &lindb, &nover, iwork);

/* Set algorithmic parameters for each block from global values */
for (k = 0; k < nb; k++) {
    lfill[k] = lfillg;
    dtol[k] = dtolg;
    pstrat[k] = pstrag;
    milu[k] = milug;
}

/* Initialize fail */
INIT_FAIL(fail);

/* Calculate factorization
 *
 * nag_sparse_nherm_precon_bdilu (f11dte). Calculates incomplete LU
 * factorization of local or overlapping diagonal blocks, mostly used
 * as incomplete LU preconditioner for complex sparse matrix.
 */
nag_sparse_nherm_precon_bdilu(n, nnz, a, la, irow, icol, nb, istb, indb,
                             lindb, lfill, dtol, pstrat, milu, ipivp,
                             ipivq, istr, idiag, &nnzc, npivm, &fail);

if (fail.code != NE_NOERROR) {

```

```

    printf("Error from nag_sparse_nherm_precon_bdilu (f11dtc).\n%s\n\n",
           fail.message);
    exit_status = -2;
    goto END;
}

/* Initialize fail */
INIT_FAIL(fail);

/* Solve Ax = b using nag_sparse_nherm_precon_bdilu_solve (f11duc)
 *
 * nag_sparse_nherm_precon_bdilu_solve (f11duc): Solves complex sparse
 * nonsymmetric linear system, using block-jacobi preconditioner
 * generated by f11dtc.
 */
nag_sparse_nherm_precon_bdilu_solve(method, n, nnz, a, la, irow, icol, nb,
                                     istb, indb, lindb, ipivp, ipivq, istr,
                                     iddiag, b, m, tol, maxitn, x, &rnorm,
                                     &itn, &fail);

if (fail.code != NE_NOERROR) {
    printf("Error from nag_sparse_nherm_precon_bdilu_solve (f11duc).\n\n%s",
           fail.message);
    exit_status = -3;
    goto END;
}

/* Print output */
printf(" Converged in %9" NAG_IFMT " iterations\n", itn);
printf(" Final residual norm = %15.6E\n", rnorm);

/* Output x */
printf(" Solution vector  x\n");
printf(" -----\n");
for (i = 0; i < n; i++) {
    printf(" ( %f, %f )\n", x[i].re, x[i].im);
}
printf("\n");

END:
NAG_FREE(b);
NAG_FREE(x);
NAG_FREE(a);
NAG_FREE(irow);
NAG_FREE(icol);
NAG_FREE(iddiag);
NAG_FREE(indb);
NAG_FREE(ipivp);
NAG_FREE(ipivq);
NAG_FREE(istr);
NAG_FREE(iwork);
NAG_FREE(dtol);
NAG_FREE(istb);
NAG_FREE(lfill);
NAG_FREE(npivm);
NAG_FREE(pstrat);
NAG_FREE(milu);

return exit_status;
}

/* ***** */
static void overlap(Integer *n, Integer *nnz, Integer *irow, Integer *icol,
                   Integer *nb, Integer *istb, Integer *indb, Integer *lindb,
                   Integer *nover, Integer *iwork)
{
    /* Purpose
     * =====
     *
     * This routine takes a set of row indices INDB defining the diagonal blocks

```

```

* to be used in nag_sparse_nherm_precon_bdilu (f11dtc) to define a block
* Jacobi or additive Schwarz preconditioner, and expands them to allow for
* NOVER levels of overlap.
*
* The pointer array ISTB is also updated accordingly, so that the returned
* values of ISTB and INDB can be passed on to
* nag_sparse_nherm_precon_bdilu (f11dtc) to define overlapping diagonal
* blocks.
*
* ----- */

/* Scalars */
Integer i, ik, ind, iover, j, k, l, n2l, nadd, row;

/* Find the number of nonzero elements in each row of the matrix A, and start
* address of each row. Store the start addresses in iwork(n,...,2*n-1).
*/

for (i = 0; i < (*n); i++) {
    iwork[i] = 0;
}

for (i = 0; i < (*nnz); i++) {
    iwork[irow[i] - 1] = iwork[irow[i] - 1] + 1;
}
iwork[(*n)] = 1;

for (i = 0; i < (*n); i++) {
    iwork[(*n) + i + 1] = iwork[(*n) + i] + iwork[i];
}

/* Loop over blocks. */
for (k = 0; k < (*nb); k++) {

    /* Initialize marker array. */
    for (j = 0; j < (*n); j++) {
        iwork[j] = 0;
    }

    /* Mark the rows already in block K in the workspace array. */
    for (l = istb[k]; l < istb[k + 1]; l++) {
        iwork[indb[l - 1] - 1] = 1;
    }

    /* Loop over levels of overlap. */
    for (iover = 1; iover <= (*nover); iover++) {

        /* Initialize counter of new row indices to be added. */
        ind = 0;

        /* Loop over the rows currently in the diagonal block. */
        for (l = istb[k]; l < istb[k + 1]; l++) {
            row = indb[l - 1];

            /* Loop over nonzero elements in row ROW. */
            for (i = iwork[(*n) + row - 1]; i < iwork[(*n) + row]; i++) {

                /* If the column index of the nonzero element is not in the
                * existing set for this block, store it to be added later, and
                * mark it in the marker array.
                */
                if (iwork[icol[i - 1] - 1] == 0) {
                    iwork[icol[i - 1] - 1] = 1;
                    iwork[2 * (*n) + 1 + ind] = icol[i - 1];
                    ind = ind + 1;
                }
            }
        }
    }

    /* Shift the indices in INDB and add the new entries for block K.
    * Change ISTB accordingly.

```

```

    */
    nadd = ind;
    if (istb[(*nb)] + nadd - 1 > (*lindb)) {
        printf("**** lindb too small, lindb = %" NAG_IFMT " ****\n", *lindb);
        exit(-1);
    }

    for (i = istb[(*nb)] - 1; i >= istb[k + 1]; i--) {
        indb[i + nadd - 1] = indb[i - 1];
    }

    n21 = 2 * (*n) + 1;
    ik = istb[k + 1] - 1;

    for (j = 0; j < nadd; j++) {
        indb[ik + j] = iwork[n21 + j];
    }

    for (j = k + 1; j < (*nb) + 1; j++) {
        istb[j] = istb[j] + nadd;
    }
}
}
return;
}

```

## 10.2 Program Data

nag\_sparse\_nherm\_precon\_bdilu\_solve (f11duc) Example Program Data

```

9          :n
33         :nnz
( 96.0, -64.0)    1    1
(-20.0,  22.0)   1    2
(-36.0,  14.0)   1    4
(-12.0,  10.0)   2    1
( 96.0, -64.0)   2    2
(-20.0,  22.0)   2    3
(-36.0,  14.0)   2    5
(-12.0,  10.0)   3    2
( 96.0, -64.0)   3    3
(-36.0,  14.0)   3    6
(-28.0,  18.0)   4    1
( 96.0, -64.0)   4    4
(-20.0,  22.0)   4    5
(-36.0,  14.0)   4    7
(-28.0,  18.0)   5    2
(-12.0,  10.0)   5    4
( 96.0, -64.0)   5    5
(-20.0,  22.0)   5    6
(-36.0,  14.0)   5    8
(-28.0,  18.0)   6    3
(-12.0,  10.0)   6    5
( 96.0, -64.0)   6    6
(-36.0,  14.0)   6    9
(-28.0,  18.0)   7    4
( 96.0, -64.0)   7    7
(-20.0,  22.0)   7    8
(-28.0,  18.0)   8    5
(-12.0,  10.0)   8    7
( 96.0, -64.0)   8    8
(-20.0,  22.0)   8    9
(-28.0,  18.0)   9    6
(-12.0,  10.0)   9    8
( 96.0, -64.0)   9    9      :a(i), irow(i), icol(i) for i=1,nnz
(100.0,  4.0)
(100.0,  4.0)
(100.0,  4.0)
(100.0,  4.0)
(100.0,  4.0)
(100.0,  4.0)

```

```
(100.0, 4.0)
(100.0, 4.0)
(100.0, 4.0)           :b(i) for i=1,n
Nag_SparseNsym_RGMRES :method
0 0.0                 :lfillg, dtolg
Nag_SparseNsym_NoPiv  :pstrag
Nag_SparseNsym_UnModFact :milug
2 1.0E-6 100          :m, tol, maxitn
3 1                   :nb, nover
```

### 10.3 Program Results

nag\_sparse\_nherm\_precon\_bdilu\_solve (f11duc) Example Program Results

```
Converged in          8 iterations
Final residual norm = 6.492468E-04
Solution vector  x
-----
( 2.203988,  1.697175 )
( 2.351101,  1.927544 )
( 1.593125,  1.436842 )
( 2.864079,  1.976215 )
( 3.068697,  2.264468 )
( 2.046736,  1.694760 )
( 2.206508,  1.324434 )
( 2.372372,  1.517040 )
( 1.625419,  1.178329 )
```

---