NAG Library Function Document

nag_zggsvd3 (f08vqc)

1 Purpose

nag_zggsvd3 (f08vqc) computes the generalized singular value decomposition (GSVD) of an m by n complex matrix A and a p by n complex matrix B.

2 Specification

```
#include <nag.h>
#include <nagf08.h>
```

```
void nag_zggsvd3 (Nag_OrderType order, Nag_ComputeUType jobu,
     Nag_ComputeVType jobv, Nag_ComputeQType jobq, Integer m, Integer n,
     Integer p, Integer *k, Integer *l, Complex a[], Integer pda,
     Complex b[], Integer pdb, double alpha[], double beta[], Complex u[],
     Integer pdu, Complex v[], Integer pdv, Complex q[], Integer pdq,
     Integer iwork[], NagError *fail)
```

3 Description

Given an m by n complex matrix A and a p by n complex matrix B, the generalized singular value decomposition is given by

$$U^{\mathrm{H}}AQ = D_1\begin{pmatrix} 0 & R \end{pmatrix}, \quad V^{\mathrm{H}}BQ = D_2\begin{pmatrix} 0 & R \end{pmatrix},$$

where U, V and Q are unitary matrices. Let l be the effective numerical rank of B and $(k+l)$ be the effective numerical rank of the matrix $\begin{pmatrix} A \\ B \end{pmatrix}$, then the first k generalized singular values are infinite and the remaining l are finite. R is a $(k+l)$ by $(k+l)$ nonsingular upper triangular matrix, D_1 and D_2 are m by $(k+l)$ and p by $(k+l)$ 'diagonal' matrices structured as follows:

if $m-k-l \geq 0$,

$$D_1 = \begin{array}{r} \\ k \\ l \\ m-k-l \end{array}\begin{array}{c} \begin{array}{cc} k & l \end{array} \\ \begin{pmatrix} I & 0 \\ 0 & C \\ 0 & 0 \end{pmatrix} \end{array}$$

$$D_2 = \begin{array}{r} \\ l \\ p-l \end{array}\begin{array}{c} \begin{array}{cc} k & l \end{array} \\ \begin{pmatrix} 0 & S \\ 0 & 0 \end{pmatrix} \end{array}$$

$$\begin{pmatrix} 0 & R \end{pmatrix} = \begin{array}{r} \\ k \\ l \end{array}\begin{array}{c} \begin{array}{ccc} n-k-l & k & l \end{array} \\ \begin{pmatrix} 0 & R_{11} & R_{12} \\ 0 & 0 & R_{22} \end{pmatrix} \end{array}$$

where

$$C = \mathrm{diag}(\alpha_{k+1}, \ldots, \alpha_{k+l}),$$
$$S = \mathrm{diag}(\beta_{k+1}, \ldots, \beta_{k+l}),$$

and

$$C^2 + S^2 = I.$$

R is stored as a submatrix of A with elements R_{ij} stored as $A_{i,n-k-l+j}$ on exit.

If $m - k - l < 0$,

$$D_1 = \begin{array}{r} \\ k \\ m-k \end{array}\!\!\begin{array}{c} \begin{array}{ccc} k & m-k & k+l-m \end{array} \\ \begin{pmatrix} I & 0 & 0 \\ 0 & C & 0 \end{pmatrix} \end{array}$$

$$D_2 = \begin{array}{r} \\ m-k \\ k+l-m \\ p-l \end{array}\!\!\begin{array}{c} \begin{array}{ccc} k & m-k & k+l-m \end{array} \\ \begin{pmatrix} 0 & S & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$\begin{pmatrix} 0 & R \end{pmatrix} = \begin{array}{r} \\ k \\ m-k \\ k+l-m \end{array}\!\!\begin{array}{c} \begin{array}{cccc} n-k-l & k & m-k & k+l-m \end{array} \\ \begin{pmatrix} 0 & R_{11} & R_{12} & R_{13} \\ 0 & 0 & R_{22} & R_{23} \\ 0 & 0 & 0 & R_{33} \end{pmatrix} \end{array}$$

where

$$C = \mathrm{diag}(\alpha_{k+1}, \ldots, \alpha_m),$$
$$S = \mathrm{diag}(\beta_{k+1}, \ldots, \beta_m),$$

and

$$C^2 + S^2 = I.$$

$\begin{pmatrix} R_{11} & R_{12} & R_{13} \\ 0 & R_{22} & R_{23} \end{pmatrix}$ is stored as a submatrix of A with R_{ij} stored as $A_{i,n-k-l+j}$, and R_{33} is stored as a submatrix of B with $(R_{33})_{ij}$ stored as $B_{m-k+i,n+m-k-l+j}$.

The function computes C, S, R and, optionally, the unitary transformation matrices U, V and Q.

In particular, if B is an n by n nonsingular matrix, then the GSVD of A and B implicitly gives the SVD of $A \times B^{-1}$:

$$AB^{-1} = U\left(D_1 D_2^{-1}\right)V^{\mathrm{H}}.$$

If $\begin{pmatrix} A \\ B \end{pmatrix}$ has orthonormal columns, then the GSVD of A and B is also equal to the CS decomposition of A and B. Furthermore, the GSVD can be used to derive the solution of the eigenvalue problem:

$$A^{\mathrm{H}}Ax = \lambda B^{\mathrm{H}}Bx.$$

In some literature, the GSVD of A and B is presented in the form

$$U^{\mathrm{H}}AX = \begin{pmatrix} 0 & D_1 \end{pmatrix}, \quad V^{\mathrm{H}}BX = \begin{pmatrix} 0 & D_2 \end{pmatrix},$$

where U and V are orthogonal and X is nonsingular, and D_1 and D_2 are 'diagonal'. The former GSVD form can be converted to the latter form by setting

$$X = Q\begin{pmatrix} I & 0 \\ 0 & R^{-1} \end{pmatrix}.$$

A two stage process is used to compute the GSVD of the matrix pair (A, B). The pair is first reduced to upper triangular form by unitary transformations using nag_zggsvp3 (f08vuc). The GSVD of the resulting upper triangular matrix pair is then performed by nag_ztgsja (f08ysc) which uses a variant of the Kogbetliantz algorithm (a cyclic Jacobi method).

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (2012) *Matrix Computations* (4th Edition) Johns Hopkins University Press, Baltimore

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **jobu** – Nag_ComputeUType *Input*

On entry: if **jobu** = Nag_AllU, the unitary matrix U is computed.

If **jobu** = Nag_NotU, U is not computed.

Constraint: **jobu** = Nag_AllU or Nag_NotU.

3: **jobv** – Nag_ComputeVType *Input*

On entry: if **jobv** = Nag_ComputeV, the unitary matrix V is computed.

If **jobv** = Nag_NotV, V is not computed.

Constraint: **jobv** = Nag_ComputeV or Nag_NotV.

4: **jobq** – Nag_ComputeQType *Input*

On entry: if **jobq** = Nag_ComputeQ, the unitary matrix Q is computed.

If **jobq** = Nag_NotQ, Q is not computed.

Constraint: **jobq** = Nag_ComputeQ or Nag_NotQ.

5: **m** – Integer *Input*

On entry: m, the number of rows of the matrix A.

Constraint: **m** ≥ 0.

6: **n** – Integer *Input*

On entry: n, the number of columns of the matrices A and B.

Constraint: **n** ≥ 0.

7: **p** – Integer *Input*

On entry: p, the number of rows of the matrix B.

Constraint: **p** ≥ 0.

8: **k** – Integer * *Output*

9: **l** – Integer * *Output*

On exit: **k** and **l** specify the dimension of the subblocks k and l as described in Section 3; $(k+l)$ is the effective numerical rank of $\begin{pmatrix} A \\ B \end{pmatrix}$.

10: **a**[dim] – Complex *Input/Output*

Note: the dimension, *dim*, of the array **a** must be at least

$\max(1, \mathbf{pda} \times \mathbf{n})$ when **order** = Nag_ColMajor;
$\max(1, \mathbf{m} \times \mathbf{pda})$ when **order** = Nag_RowMajor.

The (i, j)th element of the matrix A is stored in

> $\mathbf{a}[(j-1) \times \mathbf{pda} + i - 1]$ when $\mathbf{order} = \text{Nag\_ColMajor}$;
> $\mathbf{a}[(i-1) \times \mathbf{pda} + j - 1]$ when $\mathbf{order} = \text{Nag\_RowMajor}$.

On entry: the m by n matrix A.

On exit: contains the triangular matrix R, or part of R. See Section 3 for details.

11: **pda** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **a**.

Constraints:

> if $\mathbf{order} = \text{Nag\_ColMajor}$, $\mathbf{pda} \geq \max(1, \mathbf{m})$;
> if $\mathbf{order} = \text{Nag\_RowMajor}$, $\mathbf{pda} \geq \max(1, \mathbf{n})$.

12: $\mathbf{b}[dim]$ – Complex *Input/Output*

Note: the dimension, *dim*, of the array **b** must be at least

> $\max(1, \mathbf{pdb} \times \mathbf{n})$ when $\mathbf{order} = \text{Nag\_ColMajor}$;
> $\max(1, \mathbf{p} \times \mathbf{pdb})$ when $\mathbf{order} = \text{Nag\_RowMajor}$.

The (i, j)th element of the matrix B is stored in

> $\mathbf{b}[(j-1) \times \mathbf{pdb} + i - 1]$ when $\mathbf{order} = \text{Nag\_ColMajor}$;
> $\mathbf{b}[(i-1) \times \mathbf{pdb} + j - 1]$ when $\mathbf{order} = \text{Nag\_RowMajor}$.

On entry: the p by n matrix B.

On exit: contains the triangular matrix R if $m - k - l < 0$. See Section 3 for details.

13: **pdb** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **b**.

Constraints:

> if $\mathbf{order} = \text{Nag\_ColMajor}$, $\mathbf{pdb} \geq \max(1, \mathbf{p})$;
> if $\mathbf{order} = \text{Nag\_RowMajor}$, $\mathbf{pdb} \geq \max(1, \mathbf{n})$.

14: $\mathbf{alpha}[\mathbf{n}]$ – double *Output*

On exit: see the description of **beta**.

15: $\mathbf{beta}[\mathbf{n}]$ – double *Output*

On exit: **alpha** and **beta** contain the generalized singular value pairs of A and B, α_i and β_i;

> $\mathbf{ALPHA}(1 : \mathbf{k}) = 1$,
>
> $\mathbf{BETA}(1 : \mathbf{k}) = 0$,

and if $m - k - l \geq 0$,

> $\mathbf{ALPHA}(\mathbf{k} + 1 : \mathbf{k} + \mathbf{l}) = C$,
>
> $\mathbf{BETA}(\mathbf{k} + 1 : \mathbf{k} + \mathbf{l}) = S$,

or if $m - k - l < 0$,

> $\mathbf{ALPHA}(\mathbf{k} + 1 : \mathbf{m}) = C$,
>
> $\mathbf{ALPHA}(\mathbf{m} + 1 : \mathbf{k} + \mathbf{l}) = 0$,
>
> $\mathbf{BETA}(\mathbf{k} + 1 : \mathbf{m}) = S$,
>
> $\mathbf{BETA}(\mathbf{m} + 1 : \mathbf{k} + \mathbf{l}) = 1$, and

$\mathbf{ALPHA}(\mathbf{k}+\mathbf{l}+1:\mathbf{n}) = 0$,

$\mathbf{BETA}(\mathbf{k}+\mathbf{l}+1:\mathbf{n}) = 0$.

The notation $\mathbf{ALPHA}(\mathbf{k}:\mathbf{n})$ above refers to consecutive elements $\mathbf{alpha}[i-1]$, for $i = \mathbf{k}, \dots, \mathbf{n}$.

16: **u**[*dim*] – Complex *Output*

Note: the dimension, *dim*, of the array **u** must be at least

$\max(1, \mathbf{pdu} \times \mathbf{m})$ when **jobu** = Nag_AllU;
1 otherwise.

The (i,j)th element of the matrix U is stored in

$\mathbf{u}[(j-1) \times \mathbf{pdu} + i - 1]$ when **order** = Nag_ColMajor;
$\mathbf{u}[(i-1) \times \mathbf{pdu} + j - 1]$ when **order** = Nag_RowMajor.

On exit: if **jobu** = Nag_AllU, **u** contains the m by m unitary matrix U.

If **jobu** = Nag_NotU, **u** is not referenced.

17: **pdu** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **u**.

Constraints:

if **jobu** = Nag_AllU, $\mathbf{pdu} \geq \max(1, \mathbf{m})$;
otherwise $\mathbf{pdu} \geq 1$.

18: **v**[*dim*] – Complex *Output*

Note: the dimension, *dim*, of the array **v** must be at least

$\max(1, \mathbf{pdv} \times \mathbf{p})$ when **jobv** = Nag_ComputeV;
1 otherwise.

The (i,j)th element of the matrix V is stored in

$\mathbf{v}[(j-1) \times \mathbf{pdv} + i - 1]$ when **order** = Nag_ColMajor;
$\mathbf{v}[(i-1) \times \mathbf{pdv} + j - 1]$ when **order** = Nag_RowMajor.

On exit: if **jobv** = Nag_ComputeV, **v** contains the p by p unitary matrix V.

If **jobv** = Nag_NotV, **v** is not referenced.

19: **pdv** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **v**.

Constraints:

if **jobv** = Nag_ComputeV, $\mathbf{pdv} \geq \max(1, \mathbf{p})$;
otherwise $\mathbf{pdv} \geq 1$.

20: **q**[*dim*] – Complex *Output*

Note: the dimension, *dim*, of the array **q** must be at least

$\max(1, \mathbf{pdq} \times \mathbf{n})$ when **jobq** = Nag_ComputeQ;
1 otherwise.

The (i,j)th element of the matrix Q is stored in

$\mathbf{q}[(j-1) \times \mathbf{pdq} + i - 1]$ when **order** = Nag_ColMajor;
$\mathbf{q}[(i-1) \times \mathbf{pdq} + j - 1]$ when **order** = Nag_RowMajor.

On exit: if **jobq** = Nag_ComputeQ, **q** contains the n by n unitary matrix Q.

If **jobq** = Nag_NotQ, **q** is not referenced.

21: **pdq** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **q**.

Constraints:

if **jobq** = Nag_ComputeQ, **pdq** $\geq \max(1, \mathbf{n})$;
otherwise **pdq** ≥ 1.

22: **iwork**[**n**] – Integer *Output*

On exit: stores the sorting information. More precisely, if I is the ordered set of indices of **alpha** containing C (denote as **alpha**$[I]$, see **beta**), then the corresponding elements **iwork**$[I] - 1$ contain the swap pivots, J, that sorts I such that **alpha**$[I]$ is in descending numerical order.

The following pseudocode sorts the set I:

for $i \in I$
    $j = J_i$
    swap I_i and I_j
end

23: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_CONVERGENCE

The Jacobi-type procedure failed to converge.

NE_ENUM_INT_2

On entry, **jobq** = ⟨*value*⟩, **pdq** = ⟨*value*⟩ and **n** = ⟨*value*⟩.
Constraint: if **jobq** = Nag_ComputeQ, **pdq** $\geq \max(1, \mathbf{n})$;
otherwise **pdq** ≥ 1.

On entry, **jobu** = ⟨*value*⟩, **pdu** = ⟨*value*⟩ and **m** = ⟨*value*⟩.
Constraint: if **jobu** = Nag_AllU, **pdu** $\geq \max(1, \mathbf{m})$;
otherwise **pdu** ≥ 1.

On entry, **jobv** = ⟨*value*⟩, **pdv** = ⟨*value*⟩ and **p** = ⟨*value*⟩.
Constraint: if **jobv** = Nag_ComputeV, **pdv** $\geq \max(1, \mathbf{p})$;
otherwise **pdv** ≥ 1.

NE_INT

On entry, **m** = ⟨*value*⟩.
Constraint: **m** ≥ 0.

On entry, **n** $= \langle value \rangle$.
Constraint: **n** ≥ 0.

On entry, **p** $= \langle value \rangle$.
Constraint: **p** ≥ 0.

On entry, **pda** $= \langle value \rangle$.
Constraint: **pda** > 0.

On entry, **pdb** $= \langle value \rangle$.
Constraint: **pdb** > 0.

On entry, **pdq** $= \langle value \rangle$.
Constraint: **pdq** > 0.

On entry, **pdu** $= \langle value \rangle$.
Constraint: **pdu** > 0.

On entry, **pdv** $= \langle value \rangle$.
Constraint: **pdv** > 0.

NE_INT_2

On entry, **pda** $= \langle value \rangle$ and **m** $= \langle value \rangle$.
Constraint: **pda** $\geq \max(1, \mathbf{m})$.

On entry, **pda** $= \langle value \rangle$ and **n** $= \langle value \rangle$.
Constraint: **pda** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** $= \langle value \rangle$ and **n** $= \langle value \rangle$.
Constraint: **pdb** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** $= \langle value \rangle$ and **p** $= \langle value \rangle$.
Constraint: **pdb** $\geq \max(1, \mathbf{p})$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

The computed generalized singular value decomposition is nearly the exact generalized singular value decomposition for nearby matrices $(A + E)$ and $(B + F)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2 \text{ and } \|F\|_2 = O(\epsilon)\|B\|_2,$$

and ϵ is the ***machine precision***. See Section 4.12 of Anderson *et al.* (1999) for further details.

8 Parallelism and Performance

nag_zggsvd3 (f08vqc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag_zggsvd3 (f08vqc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Notefor your implementation for any additional implementation-specific information.

9 Further Comments

This function replaces the deprecated function nag_zggsvd (f08vnc) which used an unblocked algorithm and therefore did not make best use of level 3 BLAS functions.

The diagonal elements of the matrix R are real.

The real analogue of this function is nag_dggsvd3 (f08vcc).

10 Example

This example finds the generalized singular value decomposition

$$A = U\Sigma_1 \begin{pmatrix} 0 & R \end{pmatrix} Q^{\mathrm{H}}, \quad B = V\Sigma_2 \begin{pmatrix} 0 & R \end{pmatrix} Q^{\mathrm{H}},$$

where

$$A = \begin{pmatrix} 0.96 - 0.81i & -0.03 + 0.96i & -0.91 + 2.06i & -0.05 + 0.41i \\ -0.98 + 1.98i & -1.20 + 0.19i & -0.66 + 0.42i & -0.81 + 0.56i \\ 0.62 - 0.46i & 1.01 + 0.02i & 0.63 - 0.17i & -1.11 + 0.60i \\ 0.37 + 0.38i & 0.19 - 0.54i & -0.98 - 0.36i & 0.22 - 0.20i \\ 0.83 + 0.51i & 0.20 + 0.01i & -0.17 - 0.46i & 1.47 + 1.59i \\ 1.08 - 0.28i & 0.20 - 0.12i & -0.07 + 1.23i & 0.26 + 0.26i \end{pmatrix}$$

and

$$B = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix},$$

together with estimates for the condition number of R and the error bound for the computed generalized singular values.

The example program assumes that $m \geq n$, and would need slight modification if this is not the case.

10.1 Program Text

```
/* nag_zggsvd3 (f08vqc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nagf08.h>
#include <nagx04.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx02.h>

int main(void)
{

  /* Scalars */
  double d, eps, rcond, serrbd;
  Integer exit_status = 0, i, irank, j, k, l, m, n, p,
         pda, pdb, pdq, pdu, pdv;
  NagError fail;
  Nag_OrderType order;
```

```
  /* Arrays */
  char *clabs = 0, *rlabs = 0;
  Complex *a = 0, *b = 0, *q = 0, *u = 0, *v = 0;
  double *alpha = 0, *beta = 0;
  Integer *iwork = 0;

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J-1)*pda + I - 1]
#define B(I, J) b[(J-1)*pdb + I - 1]
  order = Nag_ColMajor;
#else
#define A(I, J) a[(I-1)*pda + J - 1]
#define B(I, J) b[(I-1)*pdb + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);

  printf("nag_zggsvd3 (f08vqc) Example Program Results\n\n");
  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif

#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[^\n] ", &m, &n, &p);
#else
  scanf("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[^\n] ", &m, &n, &p);
#endif
  if (m <= 10 && n <= 10 && p <= 10) {
    /* Allocate memory */
    if (!(clabs = NAG_ALLOC(2, char)) ||
        !(rlabs = NAG_ALLOC(2, char)) ||
        !(a = NAG_ALLOC(m * n, Complex)) ||
        !(b = NAG_ALLOC(p * n, Complex)) ||
        !(q = NAG_ALLOC(n * n, Complex)) ||
        !(u = NAG_ALLOC(m * m, Complex)) ||
        !(v = NAG_ALLOC(p * p, Complex)) ||
        !(alpha = NAG_ALLOC(n, double)) ||
        !(beta = NAG_ALLOC(n, double)) || !(iwork = NAG_ALLOC(n, Integer)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }
#ifdef NAG_COLUMN_MAJOR
    pda = m;
    pdb = p;
    pdq = n;
    pdu = m;
    pdv = p;
#else
    pda = n;
    pdb = n;
    pdq = n;
    pdu = m;
    pdv = p;
#endif
  }
  else {
    printf("m and/or n too small\n");
    goto END;
  }
  /* Read the m by n matrix A and p by n matrix B from data file */
  for (i = 1; i <= m; ++i)
    for (j = 1; j <= n; ++j)
#ifdef _WIN32
      scanf_s(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
```

```
#else
      scanf(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
#endif
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif

  for (i = 1; i <= p; ++i)
    for (j = 1; j <= n; ++j)
#ifdef _WIN32
      scanf_s(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
#else
      scanf(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
#endif
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif

  /* nag_zggsvd3 (f08vqc)
   * Compute the generalized singular value decomposition of (A, B)
   * (A = U*D1*(0 R)*(Q^H), B = V*D2*(0 R)*(Q^H), m.ge.n)
   */
  nag_zggsvd3(order, Nag_AllU, Nag_ComputeV, Nag_ComputeQ, m, n, p, &k, &l, a,
              pda, b, pdb, alpha, beta, u, pdu, v, pdv, q, pdq, iwork, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_zggsvd3 (f08vqc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }
  /* Print solution */
  irank = k + l;
  printf("Number of infinite generalized singular values (k)\n");
  printf("%5" NAG_IFMT "\n", k);
  printf("Number of finite generalized singular values (l)\n");
  printf("%5" NAG_IFMT "\n", l);
  printf("Numerical rank of ( A^T B^T)^T   (k+l)\n");
  printf("%5" NAG_IFMT "\n\n", irank);
  printf("Finite generalized singular values\n");

  for (j = k; j < irank; ++j) {
    d = alpha[j] / beta[j];
    printf("%13.4e%s", d, (j + 1) % 8 == 0 || (j + 1) == irank ? "\n" : " ");
  }
  printf("\n");

  fflush(stdout);
  nag_gen_complx_mat_print_comp(order, Nag_GeneralMatrix, Nag_NonUnitDiag,
                                m, m, u, pdu, Nag_BracketForm, "%13.4e",
                                "Unitary matrix U", Nag_IntegerLabels,
                                0, Nag_IntegerLabels, 0, 80, 0, 0, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n%s\n",
           fail.message);
    exit_status = 2;
    goto END;
  }

  printf("\n");
  fflush(stdout);
  nag_gen_complx_mat_print_comp(order, Nag_GeneralMatrix, Nag_NonUnitDiag,
                                p, p, v, pdv, Nag_BracketForm, "%13.4e",
                                "Unitary matrix V", Nag_IntegerLabels,
                                0, Nag_IntegerLabels, 0, 80, 0, 0, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n%s\n",
           fail.message);
    exit_status = 3;
```

```
    goto END;
  }

  printf("\n");
  fflush(stdout);
  nag_gen_complx_mat_print_comp(order, Nag_GeneralMatrix, Nag_NonUnitDiag,
                                n, n, q, pdq, Nag_BracketForm, "%13.4e",
                                "Unitary matrix Q", Nag_IntegerLabels,
                                0, Nag_IntegerLabels, 0, 80, 0, 0, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n%s\n",
           fail.message);
    exit_status = 4;
    goto END;
  }

  printf("\n");
  fflush(stdout);
  nag_gen_complx_mat_print_comp(order, Nag_UpperMatrix, Nag_NonUnitDiag,
                                irank, irank, &A(1, n - irank + 1), pda,
                                Nag_BracketForm, "%13.4e",
                                "Nonsingular upper triangular matrix R",
                                Nag_IntegerLabels, 0, Nag_IntegerLabels, 0,
                                80, 0, 0, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n%s\n",
           fail.message);
    exit_status = 5;
    goto END;
  }

  /* nag_ztrcon (f07tuc)
   * estimate the reciprocal condition number of R
   */
  nag_ztrcon(order, Nag_InfNorm, Nag_Upper, Nag_NonUnitDiag, irank,
             &A(1, n - irank + 1), pda, &rcond, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_ztrcon (f07tuc).\n%s\n", fail.message);
    exit_status = 6;
    goto END;
  }

  printf("\nEstimate of reciprocal condition number for R\n");
  printf("%11.1e\n\n", rcond);

  /* So long as irank = n, get the machine precision, eps, and compute the
   * approximate error bound for the computed generalized singular values
   */
  if (irank == n) {
    eps = nag_machine_precision;
    serrbd = eps / rcond;

    printf("Error estimate for the generalized singular values\n");
    printf("%11.1e\n", serrbd);
  }
  else {
    printf("(A^T B^T)^T is not of full rank\n");
  }

END:

  NAG_FREE(clabs);
  NAG_FREE(rlabs);
  NAG_FREE(a);
  NAG_FREE(b);
  NAG_FREE(q);
  NAG_FREE(u);
  NAG_FREE(v);
  NAG_FREE(alpha);
  NAG_FREE(beta);
  NAG_FREE(iwork);
```

```
  return exit_status;
}

#undef B
#undef A
```

10.2 Program Data

```
nag_zggsvd3 (f08vqc) Example Program Data

  6             4             2                         :Values of M, N and P

( 0.96,-0.81) (-0.03, 0.96) (-0.91, 2.06) (-0.05, 0.41)
(-0.98, 1.98) (-1.20, 0.19) (-0.66, 0.42) (-0.81, 0.56)
( 0.62,-0.46) ( 1.01, 0.02) ( 0.63,-0.17) (-1.11, 0.60)
( 0.37, 0.38) ( 0.19,-0.54) (-0.98,-0.36) ( 0.22,-0.20)
( 0.83, 0.51) ( 0.20, 0.01) (-0.17,-0.46) ( 1.47, 1.59)
( 1.08,-0.28) ( 0.20,-0.12) (-0.07, 1.23) ( 0.26, 0.26) :End of matrix A

( 1.00, 0.00) ( 0.00, 0.00) (-1.00, 0.00) ( 0.00, 0.00)
( 0.00, 0.00) ( 1.00, 0.00) ( 0.00, 0.00) (-1.00, 0.00) :End of matrix B
```

10.3 Program Results

```
nag_zggsvd3 (f08vqc) Example Program Results

Number of infinite generalized singular values (k)
    2
Number of finite generalized singular values (l)
    2
Numerical rank of ( A^T B^T)^T   (k+l)
    4

Finite generalized singular values
   2.0720e+00    1.1058e+00

 Unitary matrix U
                               1                             2
 1  (  -1.3038e-02,  -3.2595e-01)  (  -1.4039e-01,  -2.6167e-01)
 2  (   4.2764e-01,  -6.2582e-01)  (   8.6298e-02,  -3.8174e-02)
 3  (  -3.2595e-01,   1.6428e-01)  (   3.8163e-01,  -1.8219e-01)
 4  (   1.5906e-01,  -5.2151e-03)  (  -2.8207e-01,   1.9732e-01)
 5  (  -1.7210e-01,  -1.3038e-02)  (  -5.0942e-01,  -5.0319e-01)
 6  (  -2.6336e-01,  -2.4772e-01)  (  -1.0861e-01,   2.8474e-01)

                               3                             4
 1  (   2.5177e-01,  -7.9789e-01)  (  -5.0956e-02,  -2.1750e-01)
 2  (  -3.2188e-01,   1.6112e-01)  (   1.1979e-01,   1.6319e-01)
 3  (   1.3231e-01,  -1.4565e-02)  (  -5.0671e-01,   1.8615e-01)
 4  (   2.1598e-01,   1.8813e-01)  (  -4.0163e-01,   2.6787e-01)
 5  (   3.6488e-02,   2.0316e-01)  (   1.9271e-01,   1.5574e-01)
 6  (   1.0906e-01,  -1.2712e-01)  (  -8.8159e-02,   5.6169e-01)

                               5                             6
 1  (  -4.5947e-02,   1.4052e-04)  (  -5.2773e-02,  -2.2492e-01)
 2  (  -8.0311e-02,  -4.3605e-01)  (  -3.8117e-02,  -2.1907e-01)
 3  (   5.9714e-02,  -5.8974e-01)  (  -1.3850e-01,  -9.0941e-02)
 4  (  -4.6443e-02,   3.0864e-01)  (  -3.7354e-01,  -5.5148e-01)
 5  (   5.7843e-01,  -1.2439e-01)  (  -1.8815e-02,  -5.5686e-02)
 6  (   1.5763e-02,   4.7130e-02)  (   6.5007e-01,   4.9173e-03)

 Unitary matrix V
                               1                             2
 1  (   9.8930e-01,   1.0471e-19)  (  -1.1461e-01,   9.0250e-02)
 2  (  -1.1461e-01,  -9.0250e-02)  (  -9.8930e-01,   1.0471e-19)

 Unitary matrix Q
                               1                             2
 1  (   7.0711e-01,   0.0000e+00)  (   0.0000e+00,   0.0000e+00)
```

```
 2  (   0.0000e+00,   0.0000e+00)  (   7.0711e-01,   0.0000e+00)
 3  (   7.0711e-01,   0.0000e+00)  (   0.0000e+00,   0.0000e+00)
 4  (   0.0000e+00,   0.0000e+00)  (   7.0711e-01,   0.0000e+00)

                                3                              4
 1  (   6.9954e-01,  -1.1784e-18)  (   8.1044e-02,  -6.3817e-02)
 2  (  -8.1044e-02,  -6.3817e-02)  (   6.9954e-01,   1.1784e-18)
 3  (  -6.9954e-01,   1.1784e-18)  (  -8.1044e-02,   6.3817e-02)
 4  (   8.1044e-02,   6.3817e-02)  (  -6.9954e-01,  -1.1784e-18)

 Nonsingular upper triangular matrix R
                                1                              2
 1  (  -2.7118e+00,   0.0000e+00)  (  -1.4390e+00,  -1.0315e+00)
 2                                 (  -1.8583e+00,   0.0000e+00)
 3
 4

                                3                              4
 1  (  -7.6930e-02,   1.3613e+00)  (  -2.8137e-01,  -3.2425e-02)
 2  (  -1.0760e+00,   3.1016e-02)  (   1.3292e+00,   3.6772e-01)
 3  (   3.2537e+00,   0.0000e+00)  (  -6.3858e-17,   3.4216e-33)
 4                                 (  -2.1084e+00,   0.0000e+00)

Estimate of reciprocal condition number for R
    1.3e-01

Error estimate for the generalized singular values
    8.3e-16
```