NAG Library Function Document

nag_zgemqrt (f08aqc)

1 Purpose

nag_zgemqrt (f08aqc) multiplies an arbitrary complex matrix C by the complex unitary matrix Q from a QR factorization computed by nag_zgeqrt (f08apc).

2 Specification

```
#include <nag.h>
#include <nagf08.h>
```

```
void nag_zgemqrt (Nag_OrderType order, Nag_SideType side,
     Nag_TransType trans, Integer m, Integer n, Integer k, Integer nb,
     const Complex v[], Integer pdv, const Complex t[], Integer pdt,
     Complex c[], Integer pdc, NagError *fail)
```

3 Description

nag_zgemqrt (f08aqc) is intended to be used after a call to nag_zgeqrt (f08apc), which performs a QR factorization of a complex matrix A. The unitary matrix Q is represented as a product of elementary reflectors.

This function may be used to form one of the matrix products

$$QC, Q^{\mathrm{H}}C, CQ \text{ or } CQ^{\mathrm{H}},$$

overwriting the result on C (which may be any complex rectangular matrix).

A common application of this function is in solving linear least squares problems, as described in the f08 Chapter Introduction and illustrated in Section 10 in nag_zgeqrt (f08apc).

4 References

Golub G H and Van Loan C F (2012) *Matrix Computations* (4th Edition) Johns Hopkins University Press, Baltimore

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **side** – Nag_SideType *Input*

On entry: indicates how Q or Q^{H} is to be applied to C.

side = Nag_LeftSide
: Q or Q^{H} is applied to C from the left.

side = Nag_RightSide
: Q or Q^{H} is applied to C from the right.

Constraint: **side** = Nag_LeftSide or Nag_RightSide.

3: **trans** – Nag_TransType *Input*

On entry: indicates whether Q or Q^{H} is to be applied to C.

trans = Nag_NoTrans
Q is applied to C.

trans = Nag_ConjTrans
Q^{H} is applied to C.

Constraint: **trans** = Nag_NoTrans or Nag_ConjTrans.

4: **m** – Integer *Input*

On entry: m, the number of rows of the matrix C.

Constraint: $\mathbf{m} \geq 0$.

5: **n** – Integer *Input*

On entry: n, the number of columns of the matrix C.

Constraint: $\mathbf{n} \geq 0$.

6: **k** – Integer *Input*

On entry: k, the number of elementary reflectors whose product defines the matrix Q. Usually $\mathbf{k} = \min(m_A, n_A)$ where m_A, n_A are the dimensions of the matrix A supplied in a previous call to nag_zgeqrt (f08apc).

Constraints:

if **side** = Nag_LeftSide, $\mathbf{m} \geq \mathbf{k} \geq 0$;
if **side** = Nag_RightSide, $\mathbf{n} \geq \mathbf{k} \geq 0$.

7: **nb** – Integer *Input*

On entry: the block size used in the QR factorization performed in a previous call to nag_zgeqrt (f08apc); this value must remain unchanged from that call.

Constraints:

$\mathbf{nb} \geq 1$;
if $\mathbf{k} > 0$, $\mathbf{nb} \leq \mathbf{k}$.

8: **v**[*dim*] – const Complex *Input*

Note: the dimension, *dim*, of the array **v** must be at least

$\max(1, \mathbf{pdv} \times \mathbf{k})$ when **order** = Nag_ColMajor;
$\max(1, \mathbf{m} \times \mathbf{pdv})$ when **order** = Nag_RowMajor and **side** = Nag_LeftSide;
$\max(1, \mathbf{n} \times \mathbf{pdv})$ when **order** = Nag_RowMajor and **side** = Nag_RightSide.

On entry: details of the vectors which define the elementary reflectors, as returned by nag_zgeqrt (f08apc) in the first k columns of its array argument **a**.

9: **pdv** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **v**.

Constraints:

if **order** = Nag_ColMajor,

if **side** = Nag_LeftSide, $\mathbf{pdv} \geq \max(1, \mathbf{m})$;
if **side** = Nag_RightSide, $\mathbf{pdv} \geq \max(1, \mathbf{n})$.;

if **order** = Nag_RowMajor, $\mathbf{pdv} \geq \max(1, \mathbf{k})$.

10: **t**$[dim]$ – const Complex *Input*

Note: the dimension, *dim*, of the array **t** must be at least

max(1, **pdt** × **k**) when **order** = Nag_ColMajor;
max(1, **nb** × **pdt**) when **order** = Nag_RowMajor.

The (i, j)th element of the matrix T is stored in

t$[(j-1) \times$ **pdt** $+ i - 1]$ when **order** = Nag_ColMajor;
t$[(i-1) \times$ **pdt** $+ j - 1]$ when **order** = Nag_RowMajor.

On entry: further details of the unitary matrix Q as returned by nag_zgeqrt (f08apc). The number of blocks is $b = \left\lceil \frac{k}{\mathbf{nb}} \right\rceil$, where $k = \min(m, n)$ and each block is of order **nb** except for the last block, which is of order $k - (b-1) \times$ **nb**. For the b blocks the upper triangular block reflector factors $\boldsymbol{T}_1, \boldsymbol{T}_2, \ldots, \boldsymbol{T}_b$ are stored in the **nb** by n matrix T as $\boldsymbol{T} = [\boldsymbol{T}_1|\boldsymbol{T}_2|\ldots|\boldsymbol{T}_b]$.

11: **pdt** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **t**.

Constraints:

if **order** = Nag_ColMajor, **pdt** ≥ **nb**;
if **order** = Nag_RowMajor, **pdt** ≥ max(1, **k**).

12: **c**$[dim]$ – Complex *Input/Output*

Note: the dimension, *dim*, of the array **c** must be at least

max(1, **pdc** × **n**) when **order** = Nag_ColMajor;
max(1, **m** × **pdc**) when **order** = Nag_RowMajor.

The (i, j)th element of the matrix C is stored in

c$[(j-1) \times$ **pdc** $+ i - 1]$ when **order** = Nag_ColMajor;
c$[(i-1) \times$ **pdc** $+ j - 1]$ when **order** = Nag_RowMajor.

On entry: the m by n matrix C.

On exit: **c** is overwritten by QC or $Q^{\mathrm{H}}C$ or CQ or CQ^{H} as specified by **side** and **trans**.

13: **pdc** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **c**.

Constraints:

if **order** = Nag_ColMajor, **pdc** ≥ max(1, **m**);
if **order** = Nag_RowMajor, **pdc** ≥ max(1, **n**).

14: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_ENUM_INT_3

On entry, **side** $= \langle value \rangle$, **m** $= \langle value \rangle$, **n** $= \langle value \rangle$ and **k** $= \langle value \rangle$.
Constraint: if **side** = Nag_LeftSide, $\mathbf{m} \geq \mathbf{k} \geq 0$;
if **side** = Nag_RightSide, $\mathbf{n} \geq \mathbf{k} \geq 0$.

On entry, **side** $= \langle value \rangle$, **m** $= \langle value \rangle$, **n** $= \langle value \rangle$ and **pdv** $= \langle value \rangle$.
Constraint: if **side** = Nag_LeftSide, $\mathbf{pdv} \geq \max(1, \mathbf{m})$;
if **side** = Nag_RightSide, $\mathbf{pdv} \geq \max(1, \mathbf{n})$.

NE_INT

On entry, **m** $= \langle value \rangle$.
Constraint: $\mathbf{m} \geq 0$.

On entry, **n** $= \langle value \rangle$.
Constraint: $\mathbf{n} \geq 0$.

NE_INT_2

On entry, **nb** $= \langle value \rangle$ and **k** $= \langle value \rangle$.
Constraint: $\mathbf{nb} \geq 1$ and
if $\mathbf{k} > 0$, $\mathbf{nb} \leq \mathbf{k}$.

On entry, **pdc** $= \langle value \rangle$ and **m** $= \langle value \rangle$.
Constraint: $\mathbf{pdc} \geq \max(1, \mathbf{m})$.

On entry, **pdc** $= \langle value \rangle$ and **n** $= \langle value \rangle$.
Constraint: $\mathbf{pdc} \geq \max(1, \mathbf{n})$.

On entry, **pdt** $= \langle value \rangle$ and **k** $= \langle value \rangle$.
Constraint: $\mathbf{pdt} \geq \max(1, \mathbf{k})$.

On entry, **pdt** $= \langle value \rangle$ and **nb** $= \langle value \rangle$.
Constraint: $\mathbf{pdt} \geq \mathbf{nb}$.

On entry, **pdv** $= \langle value \rangle$ and **k** $= \langle value \rangle$.
Constraint: $\mathbf{pdv} \geq \max(1, \mathbf{k})$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

The computed result differs from the exact result by a matrix E such that

$$\|E\|_2 = O(\epsilon)\|C\|_2,$$

where ϵ is the ***machine precision***.

8 Parallelism and Performance

nag_zgemqrt (f08aqc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Notefor your implementation for any additional implementation-specific information.

9 Further Comments

The total number of real floating-point operations is approximately $8nk(2m-k)$ if **side** $=$ Nag_LeftSide and $8mk(2n-k)$ if **side** $=$ Nag_RightSide.

The real analogue of this function is nag_dgemqrt (f08acc).

10 Example

See Section 10 in nag_zgeqrt (f08apc).