

NAG Library Function Document

nag_1d_spline_intg (e02bdc)

1 Purpose

nag_1d_spline_intg (e02bdc) computes the definite integral of a cubic spline from its B-spline representation.

2 Specification

```
#include <nag.h>
#include <nage02.h>

void nag_1d_spline_intg (Nag_Spline *spline, double *integral,
                        NagError *fail)
```

3 Description

nag_1d_spline_intg (e02bdc) computes the definite integral of the cubic spline $s(x)$ between the limits $x = a$ and $x = b$, where a and b are respectively the lower and upper limits of the range over which $s(x)$ is defined. It is assumed that $s(x)$ is represented in terms of its B-spline coefficients c_i , for $i = 1, 2, \dots, \bar{n} + 3$ and (augmented) ordered knot set λ_i , for $i = 1, 2, \dots, \bar{n} + 7$, with $\lambda_i = a$, for $i = 1, 2, 3, 4$ and $\lambda_i = b$, for $i = \bar{n} + 4, \dots, \bar{n} + 7$, (see nag_1d_spline_fit_knots (e02bac)), i.e.,

$$s(x) = \sum_{i=1}^q c_i N_i(x).$$

Here $q = \bar{n} + 3$, \bar{n} is the number of intervals of the spline and $N_i(x)$ denotes the normalized B-spline of degree 3 (order 4) defined upon the knots $\lambda_i, \lambda_{i+1}, \dots, \lambda_{i+4}$.

The method employed uses the formula given in Section 3 of Cox (1975).

nag_1d_spline_intg (e02bdc) can be used to determine the definite integrals of cubic spline fits and interpolants produced by nag_1d_spline_interpolant (e01bac), nag_1d_spline_fit_knots (e02bac) and nag_1d_spline_fit (e02bec).

4 References

Cox M G (1975) An algorithm for spline interpolation *J. Inst. Math. Appl.* **15** 95–108

5 Arguments

1: **spline** – Nag_Spline *

Pointer to structure of type Nag_Spline with the following members:

n – Integer

Input

On entry: $\bar{n} + 7$, where \bar{n} is the number of intervals of the spline (which is one greater than the number of interior knots, i.e., the knots strictly within the range a to b) over which the spline is defined.

Constraint: **spline** → **n** ≥ 8.

lamda – double * *Input*

On entry: a pointer to which memory of size **spline**→**n** must be allocated. **spline**→**lamda**[*j* – 1] must be set to the value of the *j*th member of the complete set of knots, λ_j for $j = 1, 2, \dots, \bar{n} + 7$.

Constraint: the λ_j must be in nondecreasing order with **spline**→**lamda**[**spline**→**n** – 4] > **spline**→**lamda**[3] and satisfy

spline→**lamda**[0] = **spline**→**lamda**[1] = **spline**→**lamda**[2] = **spline**→**lamda**[3]

and

spline→**lamda**[**spline**→**n** – 4] = **spline**→**lamda**[**spline**→**n** – 3] =
spline→**lamda**[**spline**→**n** – 2] = **spline**→**lamda**[**spline**→**n** – 1]

c – double * *Input*

On entry: a pointer to which memory of size **spline**→**n** – 4 must be allocated. **spline**→**c** holds the coefficient c_i of the B-spline $N_i(x)$, for $i = 1, 2, \dots, \bar{n} + 3$.

2: **integral** – double * *Output*

On exit: the value of the definite integral of $s(x)$ between the limits $x = a$ and $x = b$, where $a = \lambda_4$ and $b = \lambda_{\bar{n}+4}$.

3: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_INT_ARG_LT

On entry, **spline**→**n** must not be less than 8: **spline**→**n** = *<value>*.

NE_KNOTS_CONS

On entry, the knots must satisfy the following constraints:

spline→**lamda**[**spline**→**n** – 4] > **spline**→**lamda**[3], **spline**→**lamda**[*j*] ≥ **spline**→**lamda**[*j* – 1],
 for $j = 1, 2, \dots, \mathbf{spline} \rightarrow \mathbf{n} - 1$, with equality in the cases $j = 1, 2, 3, \mathbf{spline} \rightarrow \mathbf{n} - 3,$
 $\mathbf{spline} \rightarrow \mathbf{n} - 2$ and $\mathbf{spline} \rightarrow \mathbf{n} - 1$.

7 Accuracy

The rounding errors are such that the computed value of the integral is exact for a slightly perturbed set of B-spline coefficients c_i differing in a relative sense from those supplied by no more than $2.2 \times (\bar{n} + 3) \times \mathit{machine\ precision}$.

8 Parallelism and Performance

nag_1d_spline_intg (e02bdc) is not threaded in any implementation.

9 Further Comments

Under normal usage, the call to nag_1d_spline_intg (e02bdc) will follow a call to nag_1d_spline_in
 terpolant (e01bac), nag_1d_spline_fit_knots (e02bac) or nag_1d_spline_fit (e02bec). In that case, the
 structure **spline** will have been set up correctly for input to nag_1d_spline_intg (e02bdc).

The time taken is approximately proportional to $\bar{n} + 7$.

10 Example

This example determines the definite integral over the interval $0 \leq x \leq 6$ of a cubic spline having 6 interior knots at the positions $\lambda = 1, 3, 3, 3, 4, 4$, the 8 additional knots $0, 0, 0, 0, 6, 6, 6, 6$, and the 10 B-spline coefficients $10, 12, 13, 15, 22, 26, 24, 18, 14, 12$.

The input data items (using the notation of Section 5) comprise the following values in the order indicated:

```

 $\bar{n} + 7$ 
          spline.lamda[j - 1],
for      j = 1, 2, ..., spline.n
spline.c[j - 1],    for j = 1, 2, ..., spline.n - 3

```

The example program is written in a general form that will enable the definite integral of a cubic spline having an arbitrary number of knots to be computed. Any number of datasets may be supplied. The only changes required to the program relate to the size of **spline.lamda** and the storage allocated to **spline.c** within the structure **spline**.

10.1 Program Text

```

/* nag_ld_spline_intg (e02bdc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nage02.h>

int main(void)
{
    Integer exit_status = 0, j;
    NagError fail;
    Nag_Spline spline;
    double integral;

    INIT_FAIL(fail);

    /* Initialize spline */
    spline.lamda = 0;
    spline.c = 0;

    printf("nag_ld_spline_intg (e02bdc) Example Program Results\n");
#ifdef _WIN32
    scanf_s("%*[\n]"); /* Skip heading in data file */
#else
    scanf("%*[\n]"); /* Skip heading in data file */
#endif
#ifdef _WIN32
    while (scanf_s("%" NAG_IFMT "", &(spline.n)) != EOF)
#else
    while (scanf("%" NAG_IFMT "", &(spline.n)) != EOF)
#endif
    {
        if (spline.n > 0) {
            if (!(spline.c = NAG_ALLOC(spline.n, double)) ||
                !(spline.lamda = NAG_ALLOC(spline.n, double)))
            {
                printf("Storage allocation failed. Reduce the " "size of spline.n\n");
                exit_status = 1;
                goto END;
            }
        }
    }
}

```

```

    }
  }
  else {
    printf("spline.n is out of range : spline.n = %" NAG_IFMT "\n",
           spline.n);
    exit_status = 1;
    goto END;
  }
  for (j = 0; j < spline.n; j++)
#ifdef _WIN32
    scanf_s("%lf", &(spline.lamda[j]));
#else
    scanf("%lf", &(spline.lamda[j]));
#endif
  for (j = 0; j < spline.n - 3; j++)
#ifdef _WIN32
    scanf_s("%lf", &(spline.c[j]));
#else
    scanf("%lf", &(spline.c[j]));
#endif
  /* nag_ld_spline_intg (e02bdc).
   * Evaluation of fitted cubic spline, definite integral
   */
  nag_ld_spline_intg(&spline, &integral, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_ld_spline_intg (e02bdc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }
  printf("Definite integral = %12.3e\n", integral);
  NAG_FREE(spline.c);
  NAG_FREE(spline.lamda);
}
END:
return exit_status;
}

```

10.2 Program Data

nag_ld_spline_intg (e02bdc) Example Program Data

14							
0.0	0.0	0.0	0.0	1.0	3.0	3.0	3.0
4.0	4.0	6.0	6.0	6.0	6.0		
10.0	12.0	13.0	15.0	22.0	26.0	24.0	18.0
14.0	12.0						

10.3 Program Results

nag_ld_spline_intg (e02bdc) Example Program Results

Definite integral = 1.000e+02
