

## NAG Library Function Document

### nag\_multid\_quad\_monte\_carlo\_1 (d01xbc)

#### 1 Purpose

nag\_multid\_quad\_monte\_carlo\_1 (d01xbc) evaluates an approximation to the integral of a function over a hyper-rectangular region, using a Monte–Carlo method. An approximate relative error estimate is also returned. This function is suitable for low accuracy work.

#### 2 Specification

```
#include <nag.h>
#include <nagd01.h>

void nag_multid_quad_monte_carlo_1 (Integer ndim,
    double (*f)(Integer ndim, const double x[], Nag_User *comm),
    Nag_MCMethod method, Nag_Start cont, const double a[], const double b[],
    Integer *mincls, Integer maxcls, double eps, double *finest,
    double *acc, double **comm_arr, Nag_User *comm, NagError *fail)
```

#### 3 Description

nag\_multid\_quad\_monte\_carlo\_1 (d01xbc) uses an adaptive Monte–Carlo method based on the algorithm described by Lautrup (1971). It is implemented for integrals of the form:

$$\int_{a_1}^{b_1} \int_{a_2}^{b_2} \cdots \int_{a_n}^{b_n} f(x_1, x_2, \dots, x_n) dx_n \cdots dx_2 dx_1.$$

Upon entry, unless the argument **method** = Nag\_OneIteration, the function subdivides the integration region into a number of equal volume subregions. Inside each subregion the integral and the variance are estimated by means of pseudorandom sampling. All contributions are added together to produce an estimate for the whole integral and total variance. The variance along each coordinate axis is determined and the function uses this information to increase the density and change the widths of the sub-intervals along each axis, so as to reduce the total variance. The total number of subregions is then increased by a factor of two and the program recycles for another iteration. The program stops when a desired accuracy has been reached or too many integral evaluations are needed for the next cycle.

#### 4 References

Lautrup B (1971) An adaptive multi-dimensional integration procedure *Proc. 2nd Coll. Advanced Methods in Theoretical Physics, Marseille*

#### 5 Arguments

- 1: **ndim** – Integer *Input*  
*On entry:* the number of dimensions of the integral,  $n$ .  
*Constraint:* **ndim**  $\geq 1$ .
- 2: **f** – function, supplied by the user *External Function*  
**f** must return the value of the integrand  $f$  at a given point.

The specification of **f** is:

```
double f (Integer ndim, const double x[], Nag_User *comm)
```

1:    **ndim** – Integer *Input*

*On entry:* the number of dimensions of the integral.

2:    **x[ndim]** – const double *Input*

*On entry:* the coordinates of the point at which the integrand must be evaluated.

3:    **comm** – Nag\_User \*

Pointer to a structure of type Nag\_User with the following member:

**p** – Pointer

*On entry/exit:* the pointer **comm**→**p** should be cast to the required type, e.g.,  

```
struct user *s = (struct user *)comm → p,
```

to obtain the original object's address with appropriate type. (See the argument **comm** below.)

3:    **method** – Nag\_MCMethod *Input*

*On entry:* the method to be used.

**method** = Nag\_OneIteration

The function uses only one iteration of a crude Monte–Carlo method with **maxcls** sample points.

**method** = Nag\_ManyIterations

The function subdivides the integration region into a number of equal volume subregions.

*Constraint:* **method** = Nag\_OneIteration or Nag\_ManyIterations.

4:    **cont** – Nag\_Start *Input*

*On entry:* the continuation state of the evaluation of the integrand.

**cont** = Nag\_Cold

Indicates that this is the first call to the function with the current integrand and arguments **ndim**, **a** and **b**.

**cont** = Nag\_Hot

Indicates that a previous call has been made with the same arguments **ndim**, **a** and **b** with the same integrand. Please note that **method** must not be changed.

**cont** = Nag\_Warm

Indicates that a previous call has been made with the same arguments **ndim**, **a** and **b** but that the integrand is new. Please note that **method** must not be changed.

*Constraint:* **cont** = Nag\_Cold, Nag\_Warm or Nag\_Hot.

5:    **a[ndim]** – const double *Input*

*On entry:* the lower limits of integration,  $a_i$ , for  $i = 1, 2, \dots, n$ .

6:    **b[ndim]** – const double *Input*

*On entry:* the upper limits of integration,  $b_i$ , for  $i = 1, 2, \dots, n$ .

7:    **mincls** – Integer \* *Input/Output*

*On entry:* **mincls** must be set to the minimum number of integrand evaluations to be allowed.

*Constraint:*  $0 \leq \mathbf{mincls} < \mathbf{maxcls}$ .

*On exit:* **mincls** contains the total number of integrand evaluations actually used by nag\_multid\_quad\_monte\_carlo\_1 (d01xbc).

8: **maxcls** – Integer *Input*

*On entry:* the maximum number of integrand evaluations to be allowed. In the continuation case this is the number of new integrand evaluations to be allowed. These counts do not include zero integrand values.

*Constraints:*

**maxcls** > **mincls**;  
**maxcls**  $\geq 4 \times (\text{ndim} + 1)$ .

9: **eps** – double *Input*

*On entry:* the relative accuracy required.

*Constraint:* **eps**  $\geq 0.0$ .

10: **finest** – double \* *Output*

*On exit:* the best estimate obtained for the integral.

11: **acc** – double \* *Output*

*On exit:* the estimated relative accuracy of **finest**.

12: **comm\_arr** – double \*\* *Input/Output*

*On entry:* if **cont** = Nag\_Warm or Nag\_Hot, the memory pointed to and allocated by a previous call of nag\_multid\_quad\_monte\_carlo\_1 (d01xbc) must be unchanged.

If **cont** = Nag\_Cold then appropriate memory is allocated internally by nag\_multid\_quad\_monte\_carlo\_1 (d01xbc).

*On exit:* **comm\_arr** contains information about the current sub-interval structure which could be used in later calls of nag\_multid\_quad\_monte\_carlo\_1 (d01xbc). In particular, **comm\_arr**[*j* – 1] gives the number of sub-intervals used along the *j*th coordinate axis.

When this information is no longer useful, or before a subsequent call to nag\_multid\_quad\_monte\_carlo\_1 (d01xbc) with **cont** = Nag\_Cold is made, you should free the storage contained in this pointer using the NAG macro NAG\_FREE. Note this memory will have been allocated and needs to be freed only if the error exit NE\_NOERROR or NE\_QUAD\_MAX\_INTEGRAND\_EVAL occurs. Otherwise, no memory needs to be freed.

13: **comm** – Nag\_User \*

Pointer to a structure of type Nag\_User with the following member:

**p** – Pointer

*On entry/exit:* the pointer **comm**→**p**, of type Pointer, allows you to communicate information to and from **f()**. An object of the required type should be declared, e.g., a structure, and its address assigned to the pointer **comm**→**p** by means of a cast to Pointer in the calling program, e.g., **comm.p** = (Pointer)&**s**. The type Pointer is void \*.

14: **fail** – NagError \* *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_2\_INT\_ARG\_GE

On entry, **mincls** =  $\langle value \rangle$  while **maxcls** =  $\langle value \rangle$ . These arguments must satisfy **mincls** < **maxcls**.

### NE\_2\_INT\_ARG\_LT

On entry, **maxcls** =  $\langle value \rangle$  while **ndim** =  $\langle value \rangle$ . These arguments must satisfy **maxcls**  $\geq 4 \times (\mathbf{ndim} + 1)$ .

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

### NE\_BAD\_PARAM

On entry, argument **cont** had an illegal value.

On entry, argument **method** had an illegal value.

### NE\_INT\_ARG\_LE

On entry, **mincls** =  $\langle value \rangle$ .

Constraint: **mincls** > 0.

### NE\_INT\_ARG\_LT

On entry, **ndim** =  $\langle value \rangle$ .

Constraint: **ndim**  $\geq 1$ .

### NE\_QUAD\_MAX\_INTEGRAND\_EVAL

**maxcls** was too small to obtain the required accuracy.

In this case `nag_multid_quad_monte_carlo_1` (d01xbc) returns a value of **finest** with estimated relative error **acc**, but **acc** will be greater than **eps**. This error exit may be taken before **maxcls** nonzero integrand evaluations have actually occurred, if the function calculates that the current estimates could not be improved before **maxcls** was exceeded.

### NE\_REAL\_ARG\_LT

On entry, **eps** must not be less than 0.0: **eps** =  $\langle value \rangle$ .

## 7 Accuracy

A relative error estimate is output through the argument **acc**. The confidence factor is set so that the actual error should be less than **acc** 90% of the time. If you desire a higher confidence level then a smaller value of **eps** should be used.

## 8 Parallelism and Performance

`nag_multid_quad_monte_carlo_1` (d01xbc) is not threaded in any implementation.

## 9 Further Comments

The running time for `nag_multid_quad_monte_carlo_1` (d01xbc) will usually be dominated by the time used to evaluate the integrand **f**, so the maximum time that could be used is approximately proportional to **maxcls**.

For some integrands, particularly those that are poorly behaved in a small part of the integration region, this function may terminate with a value of **acc** which is significantly smaller than the actual relative error. This should be suspected if the returned value of **mincls** is small relative to the expected

difficulty of the integral. Where this occurs, `nag_multid_quad_monte_carlo_1` (d01xbc) should be called again, but with a higher entry value of **mincls** (e.g., twice the returned value) and the results compared with those from the previous call.

## 9.1 Additional Information

The exact values of **finest** and **acc** on return will depend (within statistical limits) on the sequence of random numbers generated within this function.

If desired, you may ensure the identity or difference between runs of the results returned by this function by calling `nag_random_init_repeatable` (g05cbc) or `nag_random_init_nonrepeatable` (g05ccc) immediately prior to calling this function.

`nag_random_init_repeatable` (g05cbc) has the prototype

```
void g05cbc(Integer seed)
```

where `seed` is a scalar value used to initialize the underlying random number generator. Using the same value for `seed` will ensure that the same sequence of random values are generated and hence that the same result from this function will be obtained.

`nag_random_init_nonrepeatable` (g05ccc) has the prototype

```
void g05ccc()
```

Each time `nag_random_init_nonrepeatable` (g05ccc) is called the underlying random number generator will be reinitialized using a random seed, ensuring a different sequence of values being used. Consequently this function may return different numerical results.

## 10 Example

This example calculates the integral

$$\int_0^1 \int_0^1 \int_0^1 \int_0^1 \frac{4x_1x_3^2 \exp(2x_1x_3)}{(1+x_2+x_4)^2} dx_1 dx_2 dx_3 dx_4 = 0.575364.$$

### 10.1 Program Text

```
/* nag_multid_quad_monte_carlo_1 (d01xbc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 *
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <math.h>
#include <nagd01.h>

#ifdef __cplusplus
extern "C"
{
#endif
    static double NAG_CALL f(Integer ndim, const double x[], Nag_User *comm);
#ifdef __cplusplus
}
#endif

#define MAXCLS 20000

int main(void)
{
```

```

static Integer use_comm[1] = { 1 };
Integer exit_status = 0, k, maxcls = MAXCLS, mincls, ndim = 4;
NagError fail;
Nag_MCMethod method;
Nag_Start cont;
Nag_User comm;
double *a = 0, acc, *b = 0, *comm_arr = 0, eps, finest;

INIT_FAIL(fail);

printf("nag_multid_quad_monte_carlo_1 (d01xbc) Example Program Results\n");

/* For communication with user-supplied functions: */
comm.p = (Pointer) &use_comm;

if (ndim >= 1) {
    if (!(a = NAG_ALLOC(ndim, double)) || !(b = NAG_ALLOC(ndim, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
}
else {
    printf("Invalid ndim.\n");
    exit_status = 1;
    return exit_status;
}
for (k = 0; k < ndim; ++k) {
    a[k] = 0.0;
    b[k] = 1.0;
}
eps = 0.01;
mincls = 1000;
method = Nag_ManyIterations;
cont = Nag_Cold;

/* nag_multid_quad_monte_carlo_1 (d01xbc).
 * Multi-dimensional quadrature, using Monte Carlo method,
 * thread-safe
 */
nag_multid_quad_monte_carlo_1(ndim, f, method, cont, a, b, &mincls, maxcls,
                             eps, &finest, &acc, &comm_arr, &comm, &fail);
if (fail.code == NE_NOERROR || fail.code == NE_QUAD_MAX_INTEGRAND_EVAL) {
    if (fail.code == NE_QUAD_MAX_INTEGRAND_EVAL) {
        printf("Error from nag_multid_quad_monte_carlo_1 (d01xbc).\n%s\n",
              fail.message);
        exit_status = 2;
    }
    printf("Requested accuracy      = %11.2e\n", eps);
    printf("Estimated value          = %10.5f\n", finest);
    printf("Estimated accuracy          = %11.2e\n", acc);
    printf("Number of evaluations = %5" NAG_IFMT "\n", mincls);
}
else {
    printf("Error from nag_multid_quad_monte_carlo_1 (d01xbc).\n%s\n",
          fail.message);
    printf("%s\n", fail.message);
    exit_status = 1;
}
END:
NAG_FREE(a);
NAG_FREE(b);
/* Free memory allocated internally */
NAG_FREE(comm_arr);
return exit_status;
}

static double NAG_CALL f(Integer ndim, const double x[], Nag_User *comm)
{
    Integer *use_comm = (Integer *) comm->p;

```

```
if (use_comm[0]) {
    printf("(User-supplied callback f, first invocation.)\n");
    use_comm[0] = 0;
}

return x[0] * 4.0 * (x[2] * x[2]) * exp(x[0] * 2.0 * x[2]) /
    ((x[1] + 1.0 + x[ndim - 1]) * (x[1] + 1.0 + x[ndim - 1]));
}
```

## 10.2 Program Data

None.

## 10.3 Program Results

```
nag_multid_quad_monte_carlo_1 (d01xbc) Example Program Results
(User-supplied callback f, first invocation.)
Requested accuracy      =    1.00e-02
Estimated value        =    0.57554
Estimated accuracy     =    8.20e-03
Number of evaluations  =   1728
```

---