

NAG Library Routine Document

G13AAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

G13AAF carries out non-seasonal and seasonal differencing on a time series. Information which allows the original series to be reconstituted from the differenced series is also produced. This information is required in time series forecasting.

2 Specification

```
SUBROUTINE G13AAF (X, NX, ND, NDS, NS, XD, NXD, IFAIL)
  INTEGER          NX, ND, NDS, NS, NXD, IFAIL
  REAL (KIND=nag_wp) X(NX), XD(NX)
```

3 Description

Let $\nabla^d \nabla_s^D x_i$ be the i th value of a time series x_i , for $i = 1, 2, \dots, n$ after non-seasonal differencing of order d and seasonal differencing of order D (with period or seasonality s). In general,

$$\begin{aligned} \nabla^d \nabla_s^D x_i &= \nabla^{d-1} \nabla_s^D x_{i+1} - \nabla^{d-1} \nabla_s^D x_i & d > 0 \\ \nabla^d \nabla_s^D x_i &= \nabla^d \nabla_s^{D-1} x_{i+s} - \nabla^d \nabla_s^{D-1} x_i & D > 0 \end{aligned}$$

Non-seasonal differencing up to the required order d is obtained using

$$\begin{aligned} \nabla^1 x_i &= x_{i+1} - x_i & \text{for } i = 1, 2, \dots, (n-1) \\ \nabla^2 x_i &= \nabla^1 x_{i+1} - \nabla^1 x_i & \text{for } i = 1, 2, \dots, (n-2) \\ &\vdots \\ \nabla^d x_i &= \nabla^{d-1} x_{i+1} - \nabla^{d-1} x_i & \text{for } i = 1, 2, \dots, (n-d) \end{aligned}$$

Seasonal differencing up to the required order D is then obtained using

$$\begin{aligned} \nabla^d \nabla_s^1 x_i &= \nabla^d x_{i+s} - \nabla^d x_i & \text{for } i = 1, 2, \dots, (n-d-s) \\ \nabla^d \nabla_s^2 x_i &= \nabla^d \nabla_s^1 x_{i+s} - \nabla^d \nabla_s^1 x_i & \text{for } i = 1, 2, \dots, (n-d-2s) \\ &\vdots \\ \nabla^d \nabla_s^D x_i &= \nabla^d \nabla_s^{D-1} x_{i+s} - \nabla^d \nabla_s^{D-1} x_i & \text{for } i = 1, 2, \dots, (n-d-D \times s) \end{aligned}$$

Mathematically, the sequence in which the differencing operations are performed does not affect the final resulting series of $m = n - d - D \times s$ values.

4 References

None.

5 Arguments

1: X(NX) – REAL (KIND=nag_wp) array *Input*
On entry: the undifferenced time series, x_i , for $i = 1, 2, \dots, n$.

- 2: NX – INTEGER *Input*
On entry: n , the number of values in the undifferenced time series.
Constraint: $NX > ND + (NDS \times NS)$.
- 3: ND – INTEGER *Input*
On entry: d , the order of non-seasonal differencing.
Constraint: $ND \geq 0$.
- 4: NDS – INTEGER *Input*
On entry: D , the order of seasonal differencing.
Constraint: $NDS \geq 0$.
- 5: NS – INTEGER *Input*
On entry: s , the seasonality.
Constraints:
 if $NDS > 0$, $NS > 0$;
 if $NDS = 0$, $NS \geq 0$.
- 6: XD(NX) – REAL (KIND=nag_wp) array *Output*
On exit: the differenced values in elements 1 to NXD, and reconstitution data in the remainder of the array.
- 7: NXD – INTEGER *Output*
On exit: the number of differenced values in the array XD.
- 8: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.
 For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this argument, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**
On exit: IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, $ND < 0$,
 or $NDS < 0$,
 or $NS < 0$,
 or $NS = 0$ when $NDS > 0$.

IFAIL = 2

On entry, $NX \leq ND + (NDS \times NS)$.

IFAIL = -99

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

IFAIL = -399

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

IFAIL = -999

Dynamic memory allocation failed.

See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

The computations are believed to be stable.

8 Parallelism and Performance

G13AAF is not threaded in any implementation.

9 Further Comments

The time taken by G13AAF is approximately proportional to $(ND + NDS) \times NX$.

10 Example

This example reads in a set of data consisting of 20 observations from a time series. Non-seasonal differencing of order 2 and seasonal differencing of order 1 (with seasonality of 4) are applied to the input data, giving an output array holding 14 differenced values and 6 values which can be used to reconstitute the output array.

10.1 Program Text

```

Program g13aafe
!      G13AAF Example Program Text
!
!      Mark 26 Release. NAG Copyright 2016.
!
!      .. Use Statements ..
!      Use nag_library, Only: g13aaf, nag_wp
!      .. Implicit None Statement ..
!      Implicit None
!      .. Parameters ..
!      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
!      Integer                    :: ifail, nd, nds, ns, nx, nxd
!      .. Local Arrays ..
!      Real (Kind=nag_wp), Allocatable :: x(:), xd(:)
!      .. Executable Statements ..
!      Write (nout,*) 'G13AAF Example Program Results'
!      Write (nout,*)
!
!      Skip heading in data file
!      Read (nin,*)

```

```

!      Read in the problem size
      Read (nin,*) nx, nd, nds, ns

      Allocate (x(nx),xd(nx))

!      Read in data
      Read (nin,*) x(1:nx)

!      Perform differencing
      ifail = 0
      Call g13aaf(x,nx,nd,nds,ns,xd,nxd,ifail)

!      Display results
      Write (nout,99999) 'Non-seasonal differencing of order ', nd,      &
        ' and seasonal differencing'
      Write (nout,99999) 'of order ', nds, ' with seasonality ', ns,      &
        ' are applied'
      Write (nout,*)
      Write (nout,99998) 'The output array holds ', nx,                  &
        ' values, of which the first ', nxd, ' are differenced values'
      Write (nout,*)
      Write (nout,99997) xd(1:nx)

99999 Format (1X,A,I1,A,I1,A)
99998 Format (1X,A,I2,A,I2,A)
99997 Format (1X,5F9.1)
      End Program g13aaf

```

10.2 Program Data

G13AAF Example Program Data

```

20 2 1 4
120.0 108.0 98.0 118.0 135.0
131.0 118.0 125.0 121.0 100.0
82.0 82.0 89.0 88.0 86.0
96.0 108.0 110.0 99.0 105.0

```

10.3 Program Results

G13AAF Example Program Results

Non-seasonal differencing of order 2 and seasonal differencing
of order 1 with seasonality 4 are applied

The output array holds 20 values, of which the first 14 are differenced values

-11.0	-10.0	-8.0	4.0	12.0
-2.0	18.0	9.0	-4.0	-6.0
-5.0	-2.0	-12.0	5.0	2.0
-10.0	-13.0	17.0	6.0	105.0
