

NAG Library Routine Document

G08BAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

G08BAF performs Mood's and David's tests for dispersion differences between two independent samples of possibly unequal size.

2 Specification

```
SUBROUTINE G08BAF (X, N, N1, R, ITEST, W, V, PW, PV, IFAIL)
  INTEGER          N, N1, ITEST, IFAIL
  REAL (KIND=nag_wp) X(N), R(N), W, V, PW, PV
```

3 Description

Mood's and David's tests investigate the difference between the dispersions of two independent samples of sizes n_1 and n_2 , denoted by

$$x_1, x_2, \dots, x_{n_1}$$

and

$$x_{n_1+1}, x_{n_1+2}, \dots, x_n, \quad n = n_1 + n_2.$$

The hypothesis under test, H_0 , often called the null hypothesis, is that the dispersion difference is zero, and this is to be tested against a one- or two-sided alternative hypothesis H_1 (see below).

Both tests are based on the rankings of the sample members within the pooled sample formed by combining both samples. If there is some difference in dispersion, more of the extreme ranks will tend to be found in one sample than in the other.

Let the rank of x_i be denoted by r_i , for $i = 1, 2, \dots, n$.

(a) Mood's test.

The test statistic $W = \sum_{i=1}^{n_1} \left(r_i - \frac{n+1}{2} \right)^2$ is found.

W is the sum of squared deviations from the average rank in the pooled sample. For large n , W approaches normality, and so an approximation, p_w , to the probability of observing W not greater than the computed value, may be found.

G08BAF returns W and p_w if Mood's test is selected.

(b) David's test.

The disadvantage of Mood's test is that it assumes that the means of the two samples are equal. If this assumption is unjustified a high value of W could merely reflect the difference in means. David's test reduces this effect by using the variance of the ranks of the first sample about their mean rank, rather than the overall mean rank.

The test statistic for David's test is

$$V = \frac{1}{n_1 - 1} \sum_{i=1}^{n_1} (r_i - \bar{r})^2$$

where

$$\bar{r} = \frac{\sum_{i=1}^{n_1} r_i}{n_1}.$$

For large n , V approaches normality, enabling an approximate probability p_v to be computed, similarly to p_w .

G08BAF returns V and p_v if David's test is selected.

Suppose that a significance test of a chosen size α is to be performed (i.e., α is the probability of rejecting H_0 when H_0 is true; typically α is a small quantity such as 0.05 or 0.01).

The returned value p ($= p_v$ or p_w) can be used to perform a significance test, against various alternative hypotheses H_1 , as follows.

- (i) H_1 : dispersions are unequal. H_0 is rejected if $2 \times \min(p, 1 - p) < \alpha$.
- (ii) H_1 : dispersion of sample 1 $>$ dispersion of sample 2. H_0 is rejected if $1 - p < \alpha$.
- (iii) H_1 : dispersion of sample 2 $>$ dispersion of sample 1. H_0 is rejected if $p < \alpha$.

4 References

Cooper B E (1975) *Statistics for Experimentalists* Pergamon Press

5 Arguments

- 1: X(N) – REAL (KIND=nag_wp) array *Input*
On entry: the first n_1 elements of X must be set to the data values in the first sample, and the next n_2 ($= N - n_1$) elements to the data values in the second sample.
- 2: N – INTEGER *Input*
On entry: the total of the two sample sizes, n ($= n_1 + n_2$).
Constraint: $N > 2$.
- 3: N1 – INTEGER *Input*
On entry: the size of the first sample, n_1 .
Constraint: $1 < N1 < N$.
- 4: R(N) – REAL (KIND=nag_wp) array *Output*
On exit: the ranks r_i , assigned to the data values x_i , for $i = 1, 2, \dots, n$.
- 5: ITEST – INTEGER *Input*
On entry: the test(s) to be carried out.
ITEST = 0
Both Mood's and David's tests.
ITEST = 1
David's test only.
ITEST = 2
Mood's test only.
Constraint: ITEST = 0, 1 or 2.

- 6: W – REAL (KIND=nag_wp) Output
On exit: Mood's test statistic, W , if requested.
- 7: V – REAL (KIND=nag_wp) Output
On exit: David's test statistic, V , if requested.
- 8: PW – REAL (KIND=nag_wp) Output
On exit: the lower tail probability, p_w , corresponding to the value of W , if Mood's test was requested.
- 9: PV – REAL (KIND=nag_wp) Output
On exit: the lower tail probability, p_v , corresponding to the value of V , if David's test was requested.
- 10: IFAIL – INTEGER Input/Output
On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.
 For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this argument, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**
On exit: IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, $N \leq 2$.

IFAIL = 2

On entry, $N1 \leq 1$,
 or $N1 \geq N$.

IFAIL = 3

On entry, ITEST < 0,
 or ITEST > 2.

IFAIL = -99

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

IFAIL = -399

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

IFAIL = -999

Dynamic memory allocation failed.

See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

All computations are believed to be stable. The statistics V and W should be accurate enough for all practical uses.

8 Parallelism and Performance

G08BAF is not threaded in any implementation.

9 Further Comments

The time taken by G08BAF is small, and increases with n .

10 Example

This example is taken from page 280 of Cooper (1975). The data consists of two samples of six observations each. Both Mood's and David's test statistics and significances are computed. Note that Mood's statistic is inflated owing to the difference in location of the two samples, the median ranks differing by a factor of two.

10.1 Program Text

```

Program g08baf

!      G08BAF Example Program Text

!      Mark 26 Release. NAG Copyright 2016.

!      .. Use Statements ..
      Use nag_library, Only: g08baf, nag_wp
!      .. Implicit None Statement ..
      Implicit None
!      .. Parameters ..
      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
      Real (Kind=nag_wp)         :: pv, pw, v, w
      Integer                    :: ifail, itest, n, n1
!      .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable :: r(:), x(:)
!      .. Executable Statements ..
      Write (nout,*) 'G08BAF Example Program Results'
      Write (nout,*)

!      Skip heading in data file
      Read (nin,*)

!      Read in problem size and type of test
      Read (nin,*) n, n1, itest

      Allocate (x(n),r(n))

!      Read in data
      Read (nin,*) x(1:n)

!      Display title
      Write (nout,*) 'Mood''s test and David''s test'
      Write (nout,*)

!      Display input data

```

```

      Write (nout,*) 'Data values'
      Write (nout,*)
      Write (nout,*) 'Group 1 '
      Write (nout,99999) x(1:n1)
      Write (nout,*)
      Write (nout,*) 'Group 2 '
      Write (nout,99999) x((n1+1):n)

!      Perform test
      ifail = 0
      Call g08baf(x,n,n1,r,itest,w,v,pw,pv,ifail)

!      Display results
      Write (nout,*)
      Write (nout,99998) '      Mood''s measure = ', w, '      Significance = ', &
      pw
      Write (nout,99998) '      David''s measure = ', v, '      Significance = ', &
      pv

99999 Format (1X,8F4.0)
99998 Format (1X,A,F8.3,A,F8.4)
      End Program g08baf

```

10.2 Program Data

```

G08BAF Example Program Data
12 6 0      :: N,N1,ITEST
  6.0  9.0 12.0  4.0 10.0 11.0
  8.0  1.0  3.0  7.0  2.0  5.0 :: End of X

```

10.3 Program Results

G08BAF Example Program Results

Mood's test and David's test

Data values

Group 1
6. 9. 12. 4. 10. 11.

Group 2
8. 1. 3. 7. 2. 5.

Mood's measure =	75.500	Significance =	0.5830
David's measure =	9.467	Significance =	0.1986
