

NAG Library Routine Document

G03FAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

G03FAF performs a principal coordinate analysis also known as classical metric scaling.

2 Specification

```
SUBROUTINE G03FAF (ROOTS, N, D, NDIM, X, LDX, EVAL, WK, IWK, IFAIL)
  INTEGER          N, NDIM, LDX, IWK(5*N), IFAIL
  REAL (KIND=nag_wp) D(N*(N-1)/2), X(LDX,NDIM), EVAL(N),
  WK(N*(N+17)/2-1)
  CHARACTER(1)    ROOTS
```

3 Description

For a set of n objects a distance matrix D can be calculated such that d_{ij} is a measure of how 'far apart' are objects i and j (see G03EAF for example). Principal coordinate analysis or metric scaling starts with a distance matrix and finds points X in Euclidean space such that those points have the same distance matrix. The aim is to find a small number of dimensions, $k \ll (n-1)$, that provide an adequate representation of the distances.

The principal coordinates of the points are computed from the eigenvectors of the matrix E where $e_{ij} = -1/2(d_{ij}^2 - d_{i.}^2 - d_{.j}^2 + d_{..}^2)$ with $d_{i.}^2$ denoting the average of d_{ij}^2 over the suffix j , etc.. The eigenvectors are then scaled by multiplying by the square root of the value of the corresponding eigenvalue.

Provided that the ordered eigenvalues, λ_i , of the matrix E are all positive, $\sum_{i=1}^k \lambda_i / \sum_{i=1}^{n-1} \lambda_i$ shows how well the data is represented in k dimensions. The eigenvalues will be non-negative if E is positive semidefinite. This will be true provided d_{ij} satisfies the inequality: $d_{ij} \leq d_{ik} + d_{jk}$ for all i, j, k . If this is not the case the size of the negative eigenvalue reflects the amount of deviation from this condition and the results should be treated cautiously in the presence of large negative eigenvalues. See Krzanowski (1990) for further discussion. G03FAF provides the option for all eigenvalues to be computed so that the smallest eigenvalues can be checked.

4 References

- Chatfield C and Collins A J (1980) *Introduction to Multivariate Analysis* Chapman and Hall
- Gower J C (1966) Some distance properties of latent root and vector methods used in multivariate analysis *Biometrika* **53** 325–338
- Krzanowski W J (1990) *Principles of Multivariate Analysis* Oxford University Press

5 Arguments

- 1: ROOTS – CHARACTER(1) *Input*
- On entry:* indicates if all the eigenvalues are to be computed or just the NDIM largest.
- ROOTS = 'A'
- All the eigenvalues are computed.

ROOTS = 'L'

Only the largest NDIM eigenvalues are computed.

Constraint: ROOTS = 'A' or 'L'.

- 2: N – INTEGER *Input*
On entry: n , the number of objects in the distance matrix.
Constraint: $N > \text{NDIM}$.
- 3: $D(N \times (N - 1)/2)$ – REAL (KIND=nag_wp) array *Input*
On entry: the lower triangle of the distance matrix D stored packed by rows. That is $D((i - 1) \times (i - 2)/2 + j)$ must contain d_{ij} for $i = 2, 3, \dots, n; j = 1, 2, \dots, i - 1$.
Constraint: $D(i) \geq 0.0$, for $i = 1, 2, \dots, n(n - 1)/2$.
- 4: NDIM – INTEGER *Input*
On entry: k , the number of dimensions used to represent the data.
Constraint: $\text{NDIM} \geq 1$.
- 5: X(LDX, NDIM) – REAL (KIND=nag_wp) array *Output*
On exit: the i th row contains k coordinates for the i th point, $i = 1, 2, \dots, n$.
- 6: LDX – INTEGER *Input*
On entry: the first dimension of the array X as declared in the (sub)program from which G03FAF is called.
Constraint: $\text{LDX} \geq N$.
- 7: EVAL(N) – REAL (KIND=nag_wp) array *Output*
On exit: if ROOTS = 'A', EVAL contains the n scaled eigenvalues of the matrix E .
 If ROOTS = 'L', EVAL contains the largest k scaled eigenvalues of the matrix E .
 In both cases the eigenvalues are divided by the sum of the eigenvalues (that is, the trace of E).
- 8: $\text{WK}(N \times (N + 17)/2 - 1)$ – REAL (KIND=nag_wp) array *Workspace*
- 9: $\text{IWK}(5 \times N)$ – INTEGER array *Workspace*
- 10: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.
 For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this argument, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**
On exit: IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry $IFAIL = 0$ or -1 , explanatory error messages are output on the current error message unit (as defined by $X04AAF$).

Errors or warnings detected by the routine:

$IFAIL = 1$

On entry, $NDIM < 1$,
 or $N < NDIM$,
 or $ROOTS \neq 'A'$ or $'L'$,
 or $LDX < N$.

$IFAIL = 2$

On entry, $D(i) < 0.0$ for some i , $i = 1, 2, \dots, n(n-1)/2$,
 or all elements of $D = 0.0$.

$IFAIL = 3$

There are less than $NDIM$ eigenvalues greater than zero. Try a smaller number of dimensions ($NDIM$) or use non-metric scaling ($G03FCF$).

$IFAIL = 4$

The computation of the eigenvalues or eigenvectors has failed. Seek expert help.

$IFAIL = -99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in *How to Use the NAG Library and its Documentation* for further information.

$IFAIL = -399$

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in *How to Use the NAG Library and its Documentation* for further information.

$IFAIL = -999$

Dynamic memory allocation failed.

See Section 3.7 in *How to Use the NAG Library and its Documentation* for further information.

7 Accuracy

$G03FAF$ uses $F08JFF$ ($DSTERF$) or $F08JJF$ ($DSTEBZ$) to compute the eigenvalues and $F08JKF$ ($DSTEIN$) to compute the eigenvectors. These routines should be consulted for a discussion of the accuracy of the computations involved.

8 Parallelism and Performance

$G03FAF$ is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

$G03FAF$ makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

Alternative, non-metric, methods of scaling are provided by G03FCF.

The relationship between principal coordinates and principal components, see G03FCF, is discussed in Krzanowski (1990) and Gower (1966).

10 Example

The data, given by Krzanowski (1990), are dissimilarities between water vole populations in Europe. The first two principal coordinates are computed.

10.1 Program Text

```

Program g03fafa

!      G03FAF Example Program Text

!      Mark 26 Release. NAG Copyright 2016.

!      .. Use Statements ..
Use nag_library, Only: g03faf, nag_wp, x04caf
!      .. Implicit None Statement ..
Implicit None
!      .. Parameters ..
Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
Integer                    :: ifail, ld, ldx, liwk, lwk, n, ndim
Character (1)              :: roots
!      .. Local Arrays ..
Real (Kind=nag_wp), Allocatable :: d(:), eval(:), wk(:), x(:, :)
Integer, Allocatable       :: iwkw(:)
!      .. Executable Statements ..
Write (nout,*) 'G03FAF Example Program Results'
Write (nout,*)

!      Skip heading in data file
Read (nin,*)

!      Read in the problem size
Read (nin,*) n, ndim, roots

      ld = n*(n-1)/2
      ldx = n
      lwk = n*(n+17)/2 - 1
      liwk = 5*n
      Allocate (d(ld),x(ldx,ndim),eval(n),wk(lwk),iwkw(liwk))

!      Read in the lower triangular part of the distance matrix
Read (nin,*) d(1:ld)

!      Perform principal co-ordinate analysis
ifail = 0
Call g03faf(roots,n,d,ndim,x,ldx,eval,wk,iwkw,ifail)

!      Display results
Write (nout,*) ' Scaled Eigenvalues'
Write (nout,*)
If (roots=='L' .Or. roots=='l') Then
  Write (nout,99999) eval(1:ndim)
Else
  Write (nout,99999) eval(1:n)
End If
Write (nout,*)
Flush (nout)

```

```

        ifail = 0
        Call x04caf('General', ' ', n, ndim, x, ldx, 'Co-ordinates', ifail)

99999 Format (8F10.4)
        End Program g03fafa

```

10.2 Program Data

G03FAF Example Program Data

```

14 2 '1'
0.099
0.033 0.022
0.183 0.114 0.042
0.148 0.224 0.059 0.068
0.198 0.039 0.053 0.085 0.051
0.462 0.266 0.322 0.435 0.268 0.025
0.628 0.442 0.444 0.406 0.240 0.129 0.014
0.113 0.070 0.046 0.047 0.034 0.002 0.106 0.129
0.173 0.119 0.162 0.331 0.177 0.039 0.089 0.237 0.071
0.434 0.419 0.339 0.505 0.469 0.390 0.315 0.349 0.151 0.430
0.762 0.633 0.781 0.700 0.758 0.625 0.469 0.618 0.440 0.538 0.607
0.530 0.389 0.482 0.579 0.597 0.498 0.374 0.562 0.247 0.383 0.387 0.084
0.586 0.435 0.550 0.530 0.552 0.509 0.369 0.471 0.234 0.346 0.456 0.090 0.038

```

10.3 Program Results

G03FAF Example Program Results

Scaled Eigenvalues

```

0.7871    0.2808

```

Co-ordinates

	1	2
1	0.2408	0.2337
2	0.1137	0.1168
3	0.2394	0.0760
4	0.2129	0.0605
5	0.2495	-0.0693
6	0.1487	-0.0778
7	-0.0514	-0.1623
8	0.0115	-0.3446
9	-0.0039	0.0059
10	0.0386	-0.0089
11	-0.0421	-0.0566
12	-0.5158	0.0291
13	-0.3180	0.1501
14	-0.3238	0.0475

Example Program
Plot of Observation Number
by the First Two Principal Coordinates

