NAG Library Routine Document

G02QFF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G02QFF performs a multiple linear quantile regression, returning the parameter estimates and associated confidence limits based on an assumption of Normal, independent, identically distributed errors. G02QFF is a simplified version of G02QGF.

2 Specification

```
SUBROUTINE G02QFF (N, M, X, Y, NTAU, TAU, DF, B, BL, BU, INFO, IFAIL)

INTEGER              N, M, NTAU, INFO(NTAU), IFAIL
REAL (KIND=nag_wp) X(N,M), Y(N), TAU(NTAU), DF, B(M,NTAU), BL(M,NTAU),     &
                     BU(M,NTAU)
```

3 Description

Given a vector of n observed values, $y = \{y_i : i = 1, 2, \dots, n\}$, an $n \times p$ design matrix X, a column vector, x, of length p holding the ith row of X and a quantile $\tau \in (0, 1)$, G02QFF estimates the p-element vector β as the solution to

$$\underset{\beta \in \mathbb{R}^p}{\text{minimize}} \sum_{i=1}^{n} \rho_\tau \left(y_i - x_i^{\mathrm{T}} \beta \right) \qquad (1)$$

where ρ_τ is the piecewise linear loss function $\rho_\tau(z) = z(\tau - I(z < 0))$, and $I(z < 0)$ is an indicator function taking the value 1 if $z < 0$ and 0 otherwise.

G02QFF assumes Normal, independent, identically distributed (IID) errors and calculates the asymptotic covariance matrix from

$$\Sigma = \frac{\tau(1-\tau)}{n} (s(\tau))^2 \left(X^{\mathrm{T}} X \right)^{-1}$$

where s is the sparsity function, which is estimated from the residuals, $r_i = y_i - x_i^{\mathrm{T}} \hat{\beta}$ (see Koenker (2005)).

Given an estimate of the covariance matrix, $\hat{\Sigma}$, lower, $\hat{\beta}_L$, and upper, $\hat{\beta}_U$, limits for a 95% confidence interval are calculated for each of the p parameters, via

$$\hat{\beta}_{Li} = \hat{\beta}_i - t_{n-p,0.975} \sqrt{\hat{\Sigma}_{ii}}, \hat{\beta}_{Ui} = \hat{\beta}_i + t_{n-p,0.975} \sqrt{\hat{\Sigma}_{ii}}$$

where $t_{n-p,0.975}$ is the 97.5 percentile of the Student's t distribution with $n - k$ degrees of freedom, where k is the rank of the cross-product matrix $X^{\mathrm{T}} X$.

Further details of the algorithms used by G02QFF can be found in the documentation for G02QGF.

4 References

Koenker R (2005) *Quantile Regression* Econometric Society Monographs, Cambridge University Press, New York

5 Arguments

1: N – INTEGER *Input*

On entry: n, the number of observations in the dataset.

Constraint: $\mathrm{N} \geq 2$.

2: M – INTEGER *Input*

On entry: p, the number of variates in the model.

Constraint: $1 \leq \mathrm{M} < \mathrm{N}$.

3: X(N,M) – REAL (KIND=nag_wp) array *Input*

On entry: X, the design matrix, with the ith value for the jth variate supplied in $\mathrm{X}(i,j)$, for $i = 1,2,\ldots,\mathrm{N}$ and $j = 1,2,\ldots,\mathrm{M}$.

4: Y(N) – REAL (KIND=nag_wp) array *Input*

On entry: y, the observations on the dependent variable.

5: NTAU – INTEGER *Input*

On entry: the number of quantiles of interest.

Constraint: $\mathrm{NTAU} \geq 1$.

6: TAU(NTAU) – REAL (KIND=nag_wp) array *Input*

On entry: the vector of quantiles of interest. A separate model is fitted to each quantile.

Constraint: $\sqrt{\epsilon} < \mathrm{TAU}(l) < 1 - \sqrt{\epsilon}$ where ϵ is the ***machine precision*** returned by X02AJF, for $l = 1,2,\ldots,\mathrm{NTAU}$.

7: DF – REAL (KIND=nag_wp) *Output*

On exit: the degrees of freedom given by $n - k$, where n is the number of observations and k is the rank of the cross-product matrix $X^{\mathrm{T}}X$.

8: B(M,NTAU) – REAL (KIND=nag_wp) array *Output*

On exit: $\hat{\beta}$, the estimates of the parameters of the regression model, with $\mathrm{B}(j,l)$ containing the coefficient for the variable in column j of X, estimated for $\tau = \mathrm{TAU}(l)$.

9: BL(M,NTAU) – REAL (KIND=nag_wp) array *Output*

On exit: $\hat{\beta}_L$, the lower limit of a 95% confidence interval for $\hat{\beta}$, with $\mathrm{BL}(j,l)$ holding the lower limit associated with $\mathrm{B}(j,l)$.

10: BU(M,NTAU) – REAL (KIND=nag_wp) array *Output*

On exit: $\hat{\beta}_U$, the upper limit of a 95% confidence interval for $\hat{\beta}$, with $\mathrm{BU}(j,l)$ holding the upper limit associated with $\mathrm{B}(j,l)$.

11: INFO(NTAU) – INTEGER array *Output*

On exit: $\mathrm{INFO}(l)$ holds additional information concerning the model fitting and confidence limit calculations when $\tau = \mathrm{TAU}(l)$.

Code	**Warning**
0	Model fitted and confidence limits calculated successfully.
1	The routine did not converge whilst calculating the parameter estimates. The returned values are based on the estimate at the last iteration.

2 A singular matrix was encountered during the optimization. The model was not fitted for this value of τ.
8 The routine did not converge whilst calculating the confidence limits. The returned limits are based on the estimate at the last iteration.
16 Confidence limits for this value of τ could not be calculated. The returned upper and lower limits are set to a large positive and large negative value respectively.

It is possible for multiple warnings to be applicable to a single model. In these cases the value returned in INFO is the sum of the corresponding individual nonzero warning codes.

12: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this argument, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 11$

On entry, N $= \langle value \rangle$.
Constraint: N ≥ 2.

IFAIL $= 21$

On entry, M $= \langle value \rangle$ and N $= \langle value \rangle$.
Constraint: $1 \leq$ M $<$ N.

IFAIL $= 51$

On entry, NTAU $= \langle value \rangle$.
Constraint: NTAU ≥ 1.

IFAIL $= 61$

On entry, TAU($\langle value \rangle$) $= \langle value \rangle$ is invalid.

IFAIL $= 111$

A potential problem occurred whilst fitting the model(s).
Additional information has been returned in INFO.

IFAIL $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

IFAIL $= -399$

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

IFAIL $= -999$

Dynamic memory allocation failed.

See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

Not applicable.

8 Parallelism and Performance

G02QFF is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

G02QFF makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

Calling G02QFF is equivalent to calling G02QGF with

SORDER = 1,

INTCPT = 'N',

WEIGHT = 'U',

LDDAT = N,

setting each element of ISX to 1,

IP = M,

Interval Method = IID, and

Significance Level = 0.95.

10 Example

A quantile regression model is fitted to Engels 1857 study of household expenditure on food. The model regresses the dependent variable, household food expenditure, against household income. An intercept is included in the model by augmenting the dataset with a column of ones.

10.1 Program Text

```
    Program g02qffe

!     G02QFF Example Program Text

!     Mark 26 Release. NAG Copyright 2016.

!     .. Use Statements ..
      Use nag_library, Only: g02qff, nag_wp
!     .. Implicit None Statement ..
```

```
      Implicit None
!     .. Parameters ..
      Integer, Parameter                :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)                :: df
      Integer                           :: i, ifail, j, l, m, n, ntau
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable  :: b(:,:), bl(:,:), bu(:,:), tau(:),    &
                                           x(:,:), y(:)
      Integer, Allocatable              :: info(:)
!     .. Executable Statements ..
      Write (nout,*) 'G02QFF Example Program Results'
      Write (nout,*)

!     Skip heading in data file
      Read (nin,*)

!     Read in the problem size
      Read (nin,*) n, m, ntau

!     Read in the data
      Allocate (y(n),tau(ntau),x(n,m))
      Read (nin,*)(x(i,1:m),y(i),i=1,n)

!     Read in the quantiles required
      Read (nin,*) tau(1:ntau)

!     Allocate memory for output arrays
      Allocate (b(m,ntau),info(ntau),bl(m,ntau),bu(m,ntau))

!     Call the model fitting routine
      ifail = -1
      Call g02qff(n,m,x,y,ntau,tau,df,b,bl,bu,info,ifail)
      If (ifail/=0) Then
        If (ifail==111) Then
          Write (nout,*) 'Additional error information (INFO): ', info(1:ntau)
        Else
          Go To 100
        End If
      End If

!     Display the parameter estimates
      Do l = 1, ntau
        Write (nout,99999) 'Quantile: ', tau(l)
        Write (nout,*)
        Write (nout,*) '        Lower   Parameter   Upper'
        Write (nout,*) '        Limit   Estimate    Limit'
        Do j = 1, m
          Write (nout,99998) j, bl(j,l), b(j,l), bu(j,l)
        End Do
        Write (nout,*)
        Write (nout,*)
      End Do

100   Continue

99999 Format (1X,A,F6.3)
99998 Format (1X,I3,3(3X,F7.3))
    End Program g02qffe
```

10.2 Program Data

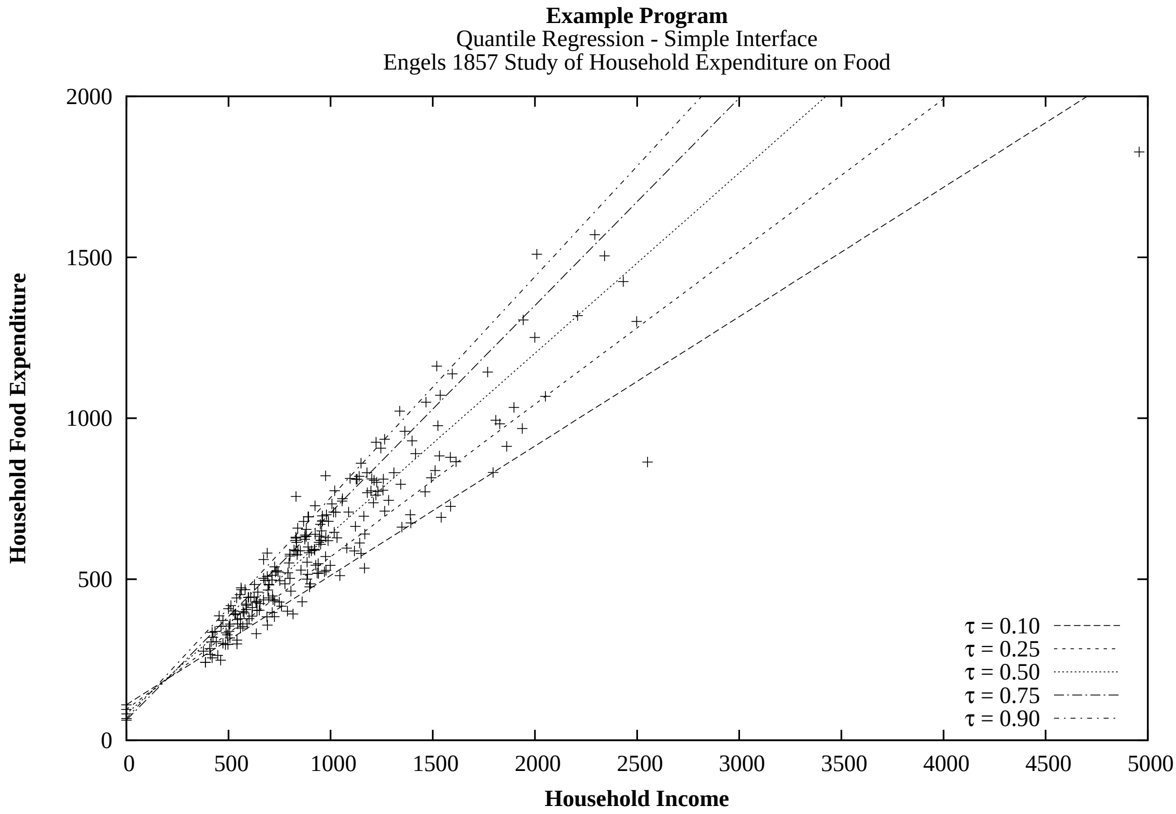
```
G02QFF Example Program Data
 235 2 5                 :: N,M,NTAU
1.0   420.1577  255.8394    1.0   800.7990  572.0807   1.0   643.3571  459.8177
1.0   541.4117  310.9587    1.0  1245.6964  907.3969   1.0  2551.6615  863.9199
1.0   901.1575  485.6800    1.0  1201.0002  811.5776   1.0  1795.3226  831.4407
1.0   639.0802  402.9974    1.0   634.4002  427.7975   1.0  1165.7734  534.7610
1.0   750.8756  495.5608    1.0   956.2315  649.9985   1.0   815.6212  392.0502
1.0   945.7989  633.7978    1.0  1148.6010  860.6002   1.0  1264.2066  934.9752
```

```
1.0    829.3979  630.7566    1.0   1768.8236 1143.4211    1.0   1095.4056   813.3081
1.0    979.1648  700.4409    1.0   2822.5330 2032.6792    1.0    447.4479   263.7100
1.0   1309.8789  830.9586    1.0    922.3548  590.6183    1.0   1178.9742   769.0838
1.0   1492.3987  815.3602    1.0   2293.1920 1570.3911    1.0    975.8023   630.5863
1.0    502.8390  338.0014    1.0    627.4726  483.4800    1.0   1017.8522   645.9874
1.0    616.7168  412.3613    1.0    889.9809  600.4804    1.0    423.8798   319.5584
1.0    790.9225  520.0006    1.0   1162.2000  696.2021    1.0    558.7767   348.4518
1.0    555.8786  452.4015    1.0   1197.0794  774.7962    1.0    943.2487   614.5068
1.0    713.4412  512.7201    1.0    530.7972  390.5984    1.0   1348.3002   662.0096
1.0    838.7561  658.8395    1.0   1142.1526  612.5619    1.0   2340.6174  1504.3708
1.0    535.0766  392.5995    1.0   1088.0039  708.7622    1.0    587.1792   406.2180
1.0    596.4408  443.5586    1.0    484.6612  296.9192    1.0   1540.9741   692.1689
1.0    924.5619  640.1164    1.0   1536.0201 1071.4627    1.0   1115.8481   588.1371
1.0    487.7583  333.8394    1.0    678.8974  496.5976    1.0   1044.6843   511.2609
1.0    692.6397  466.9583    1.0    671.8802  503.3974    1.0   1389.7929   700.5600
1.0    997.8770  543.3969    1.0    690.4683  357.6411    1.0   2497.7860  1301.1451
1.0    506.9995  317.7198    1.0    860.6948  430.3376    1.0   1585.3809   879.0660
1.0    654.1587  424.3209    1.0    873.3095  624.6990    1.0   1862.0438   912.8851
1.0    933.9193  518.9617    1.0    894.4598  582.5413    1.0   2008.8546  1509.7812
1.0    433.6813  338.0014    1.0   1148.6470  580.2215    1.0    697.3099   484.0605
1.0    587.5962  419.6412    1.0    926.8762  543.8807    1.0    571.2517   399.6703
1.0    896.4746  476.3200    1.0    839.0414  588.6372    1.0    598.3465   444.1001
1.0    454.4782  386.3602    1.0    829.4974  627.9999    1.0    461.0977   248.8101
1.0    584.9989  423.2783    1.0   1264.0043  712.1012    1.0    977.1107   527.8014
1.0    800.7990  503.3572    1.0   1937.9771  968.3949    1.0    883.9849   500.6313
1.0    502.4369  354.6389    1.0    698.8317  482.5816    1.0    718.3594   436.8107
1.0    713.5197  497.3182    1.0    920.4199  593.1694    1.0    543.8971   374.7990
1.0    906.0006  588.5195    1.0   1897.5711 1033.5658    1.0   1587.3480   726.3921
1.0    880.5969  654.5971    1.0    891.6824  693.6795    1.0   4957.8130  1827.2000
1.0    796.8289  550.7274    1.0    889.6784  693.6795    1.0    969.6838   523.4911
1.0    854.8791  528.3770    1.0   1221.4818  761.2791    1.0    419.9980   334.9998
1.0   1167.3716  640.4813    1.0    544.5991  361.3981    1.0    561.9990   473.2009
1.0    523.8000  401.3204    1.0   1031.4491  628.4522    1.0    689.5988   581.2029
1.0    670.7792  435.9990    1.0   1462.9497  771.4486    1.0   1398.5203   929.7540
1.0    377.0584  276.5606    1.0    830.4353  757.1187    1.0    820.8168   591.1974
1.0    851.5430  588.3488    1.0    975.0415  821.5970    1.0    875.1716   637.5483
1.0   1121.0937  664.1978    1.0   1337.9983 1022.3202    1.0   1392.4499   674.9509
1.0    625.5179  444.8602    1.0    867.6427  679.4407    1.0   1256.3174   776.7589
1.0    805.5377  462.8995    1.0    725.7459  538.7491    1.0   1362.8590   959.5170
1.0    558.5812  377.7792    1.0    989.0056  679.9981    1.0   1999.2552  1250.9643
1.0    884.4005  553.1504    1.0   1525.0005  977.0033    1.0   1209.4730   737.8201
1.0   1257.4989  810.8962    1.0    672.1960  561.2015    1.0   1125.0356   810.6772
1.0   2051.1789 1067.9541    1.0    923.3977  728.3997    1.0   1827.4010   983.0009
1.0   1466.3330 1049.8788    1.0    472.3215  372.3186    1.0   1014.1540   708.8968
1.0    730.0989  522.7012    1.0    590.7601  361.5210    1.0    880.3944   633.1200
1.0   2432.3910 1424.8047    1.0    831.7983  620.8006    1.0    873.7375   631.7982
1.0    940.9218  517.9196    1.0   1139.4945  819.9964    1.0    951.4432   608.6419
1.0   1177.8547  830.9586    1.0    507.5169  360.8780    1.0    473.0022   300.9999
1.0   1222.5939  925.5795    1.0    576.1972  395.7608    1.0    601.0030   377.9984
1.0   1519.5811 1162.0024    1.0    696.5991  442.0001    1.0    713.9979   397.0015
1.0    687.6638  383.4580    1.0    650.8180  404.0384    1.0    829.2984   588.5195
1.0    953.1192  621.1173    1.0    949.5802  670.7993    1.0    959.7953   681.7616
1.0    953.1192  621.1173    1.0    497.1193  297.5702    1.0   1212.9613   807.3603
1.0    953.1192  621.1173    1.0    570.1674  353.4882    1.0    958.8743   696.8011
1.0    939.0418  548.6002    1.0    724.7306  383.9376    1.0   1129.4431   811.1962
1.0   1283.4025  745.2353    1.0    408.3399  284.8008    1.0   1943.0419  1305.7201
1.0   1511.5789  837.8005    1.0    638.6713  431.1000    1.0    539.6388   442.0001
1.0   1342.5821  795.3402    1.0   1225.7890  801.3518    1.0    463.5990   353.6013
1.0    511.7980  418.5976    1.0    715.3701  448.4513    1.0    562.6400   468.0008
1.0    689.7988  508.7974    1.0    800.4708  577.9111    1.0    736.7584   526.7573
1.0   1532.3074  883.2780    1.0    975.5974  570.5210    1.0   1415.4461   890.2390
1.0   1056.0808  742.5276    1.0   1613.7565  865.3205    1.0   2208.7897  1318.8033
1.0    387.3195  242.3202    1.0    608.5019  444.5578    1.0    636.0009   331.0005
1.0    387.3195  242.3202    1.0    958.6634  680.4198    1.0    759.4010   416.4015
1.0    410.9987  266.0010    1.0    835.9426  576.2779    1.0   1078.8382   596.8406
1.0    499.7510  408.4992    1.0   1024.8177  708.4787    1.0    748.6413   429.0399
1.0    832.7554  614.7588    1.0   1006.4353  734.2356    1.0    987.6417   619.6408
1.0    614.9986  385.3184    1.0    726.0000  433.0010    1.0    788.0961   400.7990
1.0    887.4658  515.6200    1.0    494.4174  327.4188    1.0   1020.0225   775.0209
```

```
1.0  1595.1611 1138.1620    1.0   776.5958  485.5198   1.0  1230.9235  772.7611
1.0  1807.9520  993.9630    1.0   415.4407  305.4390   1.0   440.5174  306.5191
1.0   541.2006  299.1993    1.0   581.3599  468.0008   1.0   743.0772  522.6019
1.0  1057.6767  750.3202 :: End of X,Y (in three  set of columns)
0.10 0.25 0.50 0.75 0.90 :: TAU
```

10.3 Program Results

```
 G02QFF Example Program Results

 Quantile:  0.100

         Lower   Parameter   Upper
         Limit   Estimate    Limit
   1    74.946   110.142   145.337
   2     0.370     0.402     0.433


 Quantile:  0.250

         Lower   Parameter   Upper
         Limit   Estimate    Limit
   1    64.232    95.483   126.735
   2     0.446     0.474     0.502


 Quantile:  0.500

         Lower   Parameter   Upper
         Limit   Estimate    Limit
   1    55.399    81.482   107.566
   2     0.537     0.560     0.584


 Quantile:  0.750

         Lower   Parameter   Upper
         Limit   Estimate    Limit
   1    41.372    62.396    83.421
   2     0.625     0.644     0.663


 Quantile:  0.900

         Lower   Parameter   Upper
         Limit   Estimate    Limit
   1    26.829    67.351   107.873
   2     0.650     0.686     0.723
```


Example Program
Quantile Regression - Simple Interface
Engels 1857 Study of Household Expenditure on Food
Household Food Expenditure
Household Income
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
τ = 0.10
τ = 0.25
τ = 0.50
τ = 0.75
τ = 0.90