

# NAG Library Routine Document

## G02CCF

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

### 1 Purpose

G02CCF performs a simple linear regression with dependent variable  $y$  and independent variable  $x$ , omitting cases involving missing values.

### 2 Specification

```
SUBROUTINE G02CCF (N, X, Y, XMISS, YMISS, RESULT, IFAIL)
  INTEGER          N, IFAIL
  REAL (KIND=nag_wp) X(N), Y(N), XMISS, YMISS, RESULT(21)
```

### 3 Description

G02CCF fits a straight line of the form

$$y = a + bx$$

to those of the data points

$$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$$

that do not include missing values, such that

$$y_i = a + bx_i + e_i$$

for those  $(x_i, y_i)$ ,  $i = 1, 2, \dots, n$  ( $n > 2$ ) which do not include missing values.

The routine eliminates all pairs of observations  $(x_i, y_i)$  which contain a missing value for either  $x$  or  $y$ , and then calculates the regression coefficient,  $b$ , the regression constant,  $a$ , and various other statistical quantities, by minimizing the sum of the  $e_i^2$  over those cases remaining in the calculations.

The input data consists of the  $n$  pairs of observations  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$  on the independent variable  $x$  and the dependent variable  $y$ .

In addition two values,  $xm$  and  $ym$ , are given which are considered to represent missing observations for  $x$  and  $y$  respectively. (See Section 7).

Let  $w_i = 0$  if the  $i$ th observation of either  $x$  or  $y$  is missing, i.e., if  $x_i = xm$  and/or  $y_i = ym$ ; and  $w_i = 1$  otherwise, for  $i = 1, 2, \dots, n$ .

The quantities calculated are:

(a) Means:

$$\bar{x} = \frac{\sum_{i=1}^n w_i x_i}{\sum_{i=1}^n w_i}; \quad \bar{y} = \frac{\sum_{i=1}^n w_i y_i}{\sum_{i=1}^n w_i}.$$

(b) Standard deviations:

$$s_x = \sqrt{\frac{\sum_{i=1}^n w_i (x_i - \bar{x})^2}{\sum_{i=1}^n w_i - 1}}; \quad s_y = \sqrt{\frac{\sum_{i=1}^n w_i (y_i - \bar{y})^2}{\sum_{i=1}^n w_i - 1}}.$$

(c) Pearson product-moment correlation coefficient:

$$r = \frac{\sum_{i=1}^n w_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n w_i (x_i - \bar{x})^2 \sum_{i=1}^n w_i (y_i - \bar{y})^2}}.$$

(d) The regression coefficient,  $b$ , and the regression constant,  $a$ :

$$b = \frac{\sum_{i=1}^n w_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n w_i (x_i - \bar{x})^2}, \quad a = \bar{y} - b\bar{x}.$$

(e) The sum of squares attributable to the regression,  $SSR$ , the sum of squares of deviations about the regression,  $SSD$ , and the total sum of squares,  $SST$ :

$$SST = \sum_{i=1}^n w_i (y_i - \bar{y})^2; \quad SSD = \sum_{i=1}^n w_i (y_i - a - bx_i)^2; \quad SSR = SST - SSD.$$

(f) The degrees of freedom attributable to the regression,  $DFR$ , the degrees of freedom of deviations about the regression,  $DFD$ , and the total degrees of freedom,  $DFT$ :

$$DFT = \sum_{i=1}^n w_i - 1; \quad DFD = \sum_{i=1}^n w_i - 2; \quad DFR = 1.$$

(g) The mean square attributable to the regression,  $MSR$ , and the mean square of deviations about the regression,  $MSD$ :

$$MSR = SSR/DFR; \quad MSD = SSD/DFD.$$

(h) The  $F$  value for the analysis of variance:

$$F = MSR/MSD.$$

(i) The standard error of the regression coefficient,  $se(b)$ , and the standard error of the regression constant,  $se(a)$ :

$$se(b) = \sqrt{\frac{MSD}{\sum_{i=1}^n w_i (x_i - \bar{x})^2}}; \quad se(a) = \sqrt{MSD \left( \frac{1}{\sum_{i=1}^n w_i} + \frac{\bar{x}^2}{\sum_{i=1}^n w_i (x_i - \bar{x})^2} \right)}.$$

(j) The  $t$  value for the regression coefficient,  $t(b)$ , and the  $t$  value for the regression constant,  $t(a)$ :

$$t(b) = \frac{b}{se(b)}; \quad t(a) = \frac{a}{se(a)}.$$

(k) The number of observations used in the calculations:

$$n_c = \sum_{i=1}^n w_i.$$

## 4 References

Draper N R and Smith H (1985) *Applied Regression Analysis* (2nd Edition) Wiley

## 5 Arguments

- 1: N – INTEGER *Input*  
*On entry:*  $n$ , the number of pairs of observations.  
*Constraint:*  $N > 2$ .
- 2: X(N) – REAL (KIND=nag\_wp) array *Input*  
*On entry:*  $X(i)$  must contain  $x_i$ , for  $i = 1, 2, \dots, n$ .
- 3: Y(N) – REAL (KIND=nag\_wp) array *Input*  
*On entry:*  $Y(i)$  must contain  $y_i$ , for  $i = 1, 2, \dots, n$ .
- 4: XMISS – REAL (KIND=nag\_wp) *Input*  
*On entry:* the value  $xm$  which is to be taken as the missing value for the variable  $x$ . See Section 7.
- 5: YMISS – REAL (KIND=nag\_wp) *Input*  
*On entry:* the value  $ym$  which is to be taken as the missing value for the variable  $y$ . See Section 7.
- 6: RESULT(21) – REAL (KIND=nag\_wp) array *Output*  
*On exit:* the following information:
- RESULT(1)  $\bar{x}$ , the mean value of the independent variable,  $x$ ;  
 RESULT(2)  $\bar{y}$ , the mean value of the dependent variable,  $y$ ;  
 RESULT(3)  $s_x$ , the standard deviation of the independent variable,  $x$ ;  
 RESULT(4)  $s_y$ , the standard deviation of the dependent variable,  $y$ ;  
 RESULT(5)  $r$ , the Pearson product-moment correlation between the independent variable  $x$  and the dependent variable  $y$   
 RESULT(6)  $b$ , the regression coefficient;  
 RESULT(7)  $a$ , the regression constant;  
 RESULT(8)  $se(b)$ , the standard error of the regression coefficient;  
 RESULT(9)  $se(a)$ , the standard error of the regression constant;  
 RESULT(10)  $t(b)$ , the  $t$  value for the regression coefficient;  
 RESULT(11)  $t(a)$ , the  $t$  value for the regression constant;  
 RESULT(12)  $SSR$ , the sum of squares attributable to the regression;  
 RESULT(13)  $DFR$ , the degrees of freedom attributable to the regression;  
 RESULT(14)  $MSR$ , the mean square attributable to the regression;  
 RESULT(15)  $F$ , the  $F$  value for the analysis of variance;  
 RESULT(16)  $SSD$ , the sum of squares of deviations about the regression;  
 RESULT(17)  $DFD$ , the degrees of freedom of deviations about the regression;  
 RESULT(18)  $MSD$ , the mean square of deviations about the regression;  
 RESULT(19)  $SST$ , the total sum of squares;  
 RESULT(20)  $DFT$ , the total degrees of freedom;  
 RESULT(21)  $n_c$ , the number of observations used in the calculations.
- 7: IFAIL – INTEGER *Input/Output*  
*On entry:* IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value  $-1$  or  $1$  is recommended. If the output of error messages is undesirable, then the value  $1$  is recommended. Otherwise, if you are not familiar with this argument, the recommended value is  $0$ . **When the value  $-1$  or  $1$  is used it is essential to test the value of IFAIL on exit.**

*On exit:* IFAIL =  $0$  unless the routine detects an error or a warning has been flagged (see Section 6).

## 6 Error Indicators and Warnings

If on entry IFAIL =  $0$  or  $-1$ , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL =  $1$

On entry,  $N \leq 2$ .

IFAIL =  $2$

After observations with missing values were omitted, two or fewer cases remained.

IFAIL =  $3$

After observations with missing values were omitted, all remaining values of at least one of the variables  $x$  and  $y$  were identical.

IFAIL =  $-99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

IFAIL =  $-399$

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

IFAIL =  $-999$

Dynamic memory allocation failed.

See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

## 7 Accuracy

G02CCF does not use *additional precision* arithmetic for the accumulation of scalar products, so there may be a loss of significant figures for large  $n$ .

You are warned of the need to exercise extreme care in your selection of missing values. G02CCF treats all values in the inclusive range  $(1 \pm 0.1^{(X02BEF-2)}) \times xm_j$ , where  $xm_j$  is the missing value for variable  $j$  specified in XMISS.

You must therefore ensure that the missing value chosen for each variable is sufficiently different from all valid values for that variable so that none of the valid values fall within the range indicated above.

If, in calculating  $F$  or  $t(a)$  (see Section 3), the numbers involved are such that the result would be outside the range of numbers which can be stored by the machine, then the answer is set to the largest quantity which can be stored as a real variable, by means of a call to X02ALF.

## 8 Parallelism and Performance

G02CCF is not threaded in any implementation.

## 9 Further Comments

The time taken by G02CCF depends on  $n$  and the number of missing observations.

The routine uses a two-pass algorithm.

## 10 Example

This example reads in eight observations on each of two variables, and then performs a simple linear regression with the first variable as the independent variable, and the second variable as the dependent variable, omitting cases involving missing values (0.0 for the first variable, 99.0 for the second). Finally the results are printed.

### 10.1 Program Text

```

Program g02ccfe

!      G02CCF Example Program Text

!      Mark 26 Release. NAG Copyright 2016.

!      .. Use Statements ..
!      Use nag_library, Only: g02ccf, nag_wp
!      .. Implicit None Statement ..
!      Implicit None
!      .. Parameters ..
!      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
!      Real (Kind=nag_wp)          :: xmiss, ymiss
!      Integer                     :: i, ifail, n
!      .. Local Arrays ..
!      Real (Kind=nag_wp)          :: reslt(21)
!      Real (Kind=nag_wp), Allocatable :: x(:), y(:)
!      .. Executable Statements ..
!      Write (nout,*) 'G02CCF Example Program Results'
!      Write (nout,*)

!      Skip heading in data file
!      Read (nin,*)

!      Read in problem size
!      Read (nin,*) n

!      Allocate (x(n),y(n))

!      Read in data
!      Read (nin,*)(x(i),y(i),i=1,n)

!      Read in missing value flags
!      Read (nin,*) xmiss, ymiss

!      Display data
!      Write (nout,*) ' Case      Independent      Dependent'
!      Write (nout,*) 'number      variable      variable'
!      Write (nout,*)
!      Write (nout,99999)(i,x(i),y(i),i=1,n)
!      Write (nout,*)

!      Fit linear regression model
!      ifail = 0
!      Call g02ccf(n,x,y,xmiss,ymiss,reslt,ifail)

!      Write (nout,99998) 'Mean of independent variable      = ',      &

```

```

      reslt(1)
Write (nout,99998) 'Mean of   dependent variable           = ',      &
      reslt(2)
Write (nout,99998) 'Standard deviation of independent variable = ',      &
      reslt(3)
Write (nout,99998) 'Standard deviation of   dependent variable = ',      &
      reslt(4)
Write (nout,99998) 'Correlation coefficient                 = ',      &
      reslt(5)
Write (nout,*)
Write (nout,99998) 'Regression coefficient                 = ',      &
      reslt(6)
Write (nout,99998) 'Standard error of coefficient         = ',      &
      reslt(8)
Write (nout,99998) 't-value for coefficient               = ',      &
      reslt(10)
Write (nout,*)
Write (nout,99998) 'Regression constant                   = ',      &
      reslt(7)
Write (nout,99998) 'Standard error of constant           = ',      &
      reslt(9)
Write (nout,99998) 't-value for constant                 = ',      &
      reslt(11)
Write (nout,*)
Write (nout,*) 'Analysis of regression table :-'
Write (nout,*)
Write (nout,*)
      '          Source          Sum of squares  D.F.    Mean square    F-value' &
Write (nout,*)
Write (nout,99997) 'Due to regression', reslt(12:15)
Write (nout,99997) 'About regression', reslt(16:18)
Write (nout,99997) 'Total          ', reslt(19:20)
Write (nout,*)
Write (nout,99996) 'Number of cases used = ', reslt(21)

99999 Format (1X,I4,2F15.4)
99998 Format (1X,A,F8.4)
99997 Format (1X,A,F14.4,F8.0,2F14.4)
99996 Format (1X,A,F3.0)
      End Program g02ccfe

```

## 10.2 Program Data

```

G02CCF Example Program Data
8          :: N
1.0       20.0
0.0       15.5
4.0       28.3
7.5       45.0
2.5       24.5
0.0       10.0
10.0      99.0
5.0       31.2 :: End of X, Y
0.0       99.0 :: XMISS, YMISS

```

## 10.3 Program Results

G02CCF Example Program Results

Case number	Independent variable	Dependent variable
1	1.0000	20.0000
2	0.0000	15.5000
3	4.0000	28.3000
4	7.5000	45.0000
5	2.5000	24.5000
6	0.0000	10.0000
7	10.0000	99.0000
8	5.0000	31.2000

Mean of independent variable = 4.0000  
 Mean of dependent variable = 29.8000  
 Standard deviation of independent variable = 2.4749  
 Standard deviation of dependent variable = 9.4787  
 Correlation coefficient = 0.9799  
  
 Regression coefficient = 3.7531  
 Standard error of coefficient = 0.4409  
 t-value for coefficient = 8.5128  
  
 Regression constant = 14.7878  
 Standard error of constant = 2.0155  
 t-value for constant = 7.3370

Analysis of regression table :-

Source	Sum of squares	D.F.	Mean square	F-value
Due to regression	345.0940	1.	345.0940	72.4682
About regression	14.2860	3.	4.7620	
Total	359.3800	4.		

Number of cases used = 5.

---