NAG Library Routine Document

G01ECF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G01ECF returns the lower or upper tail probability for the χ^2-distribution with real degrees of freedom, via the routine name.

2 Specification

```
FUNCTION G01ECF (TAIL, X, DF, IFAIL)
REAL (KIND=nag_wp) G01ECF
INTEGER            IFAIL
REAL (KIND=nag_wp) X, DF
CHARACTER(1)       TAIL
```

3 Description

The lower tail probability for the χ^2-distribution with ν degrees of freedom, $P(X \leq x : \nu)$ is defined by:

$$P(X \leq x : \nu) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} \int_{0.0}^{x} X^{\nu/2-1} e^{-X/2}\, dX, \quad x \geq 0, \nu > 0.$$

To calculate $P(X \leq x : \nu)$ a transformation of a gamma distribution is employed, i.e., a χ^2-distribution with ν degrees of freedom is equal to a gamma distribution with scale parameter 2 and shape parameter $\nu/2$.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

5 Arguments

1: TAIL – CHARACTER(1) *Input*

On entry: indicates whether the upper or lower tail probability is required.

TAIL = 'L'
The lower tail probability is returned, i.e., $P(X \leq x : \nu)$.

TAIL = 'U'
The upper tail probability is returned, i.e., $P(X \geq x : \nu)$.

Constraint: TAIL = 'L' or 'U'.

2: X – REAL (KIND=nag_wp) *Input*

On entry: x, the value of the χ^2 variate with ν degrees of freedom.

Constraint: X $\geq$ 0.0.

3: DF – REAL (KIND=nag_wp) *Input*

On entry: ν, the degrees of freedom of the χ^2-distribution.

Constraint: DF > 0.0.

4: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output arguments may be useful even if IFAIL $\neq 0$ on exit, the recommended value is -1. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Note: G01ECF may return useful information for one or more of the following detected errors or warnings.

Errors or warnings detected by the routine:

If IFAIL $= 1$, 2 or 3 on exit, then G01ECF returns 0.0.

IFAIL $= 1$

On entry, TAIL $\neq$ 'L' or 'U'.

IFAIL $= 2$

On entry, X < 0.0.

IFAIL $= 3$

On entry, DF ≤ 0.0.

IFAIL $= 4$

The solution has failed to converge while calculating the gamma variate. The result returned should represent an approximation to the solution.

IFAIL $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

IFAIL $= -399$

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

IFAIL $= -999$

Dynamic memory allocation failed.

See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

A relative accuracy of five significant figures is obtained in most cases.

8 Parallelism and Performance

G01ECF is not threaded in any implementation.

9 Further Comments

For higher accuracy the transformation described in Section 3 may be used with a direct call to S14BAF.

10 Example

Values from various χ^2-distributions are read, the lower tail probabilities calculated, and all these values printed out, until the end of data is reached.

10.1 Program Text

```
    Program g01ecfe

!     G01ECF Example Program Text

!     Mark 26 Release. NAG Copyright 2016.

!     .. Use Statements ..
      Use nag_library, Only: g01ecf, nag_wp
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: df, prob, x
      Integer                          :: ifail
      Character (1)                    :: tail
!     .. Executable Statements ..
      Write (nout,*) 'G01ECF Example Program Results'
      Write (nout,*)

!     Skip heading in data file
      Read (nin,*)

!     Display titles
      Write (nout,*) ' TAIL     X       DF      Probability'
      Write (nout,*)

d_lp: Do
        Read (nin,*,Iostat=ifail) tail, x, df
        If (ifail/=0) Then
          Exit d_lp
        End If

!       Calculate probability
        ifail = -1
        prob = g01ecf(tail,x,df,ifail)
        If (ifail/=0) Then
          If (ifail/=4) Then
            Exit d_lp
          End If
        End If

!       Display results
```

```
        Write (nout,99999) tail, x, df, prob
      End Do d_lp

99999 Format (3X,A1,4X,F6.3,F8.1,7X,F7.4)
    End Program g01ecfe
```

10.2 Program Data

```
G01ECF Example Program Data
'L'  8.26   20.0 : TAIL X DF
'L'  6.2     7.5
'L' 55.76   45.0
```

10.3 Program Results

```
 G01ECF Example Program Results

  TAIL     X       DF      Probability

   L     8.260    20.0        0.0100
   L     6.200     7.5        0.4279
   L    55.760    45.0        0.8694
```