

## NAG Library Routine Document

### F16RBF

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

## 1 Purpose

F16RBF calculates the value of the 1-norm, the  $\infty$ -norm, the Frobenius norm or the maximum absolute value of the elements of a real  $m$  by  $n$  band matrix stored in banded form.

It can also be used to compute the value of the 2-norm of a row  $n$ -vector or a column  $m$ -vector.

## 2 Specification

```
FUNCTION F16RBF (INORM, M, N, KL, KU, AB, LDAB)
REAL (KIND=nag_wp) F16RBF
INTEGER          INORM, M, N, KL, KU, LDAB
REAL (KIND=nag_wp) AB(LDAB,*)
```

## 3 Description

Given a real  $m$  by  $n$  banded matrix,  $A$ , F16RBF calculates one of the values given by

$$\|A\|_1 = \max_j \sum_{i=1}^m |a_{ij}| \quad (\text{the 1-norm of } A),$$

$$\|A\|_\infty = \max_i \sum_{j=1}^n |a_{ij}| \quad (\text{the } \infty\text{-norm of } A),$$

$$\|A\|_F = \left( \sum_{i=1}^m \sum_{j=1}^n |a_{ij}|^2 \right)^{1/2} \quad (\text{the Frobenius norm of } A), \quad \text{or}$$

$$\max_{i,j} |a_{ij}| \quad (\text{the maximum absolute element value of } A).$$

If  $m$  or  $n$  is 1 then additionally F16RBF can calculate the value  $\|A\|_2 = \sqrt{\sum a_i^2}$  (the 2-norm of  $A$ ).

## 4 References

Basic Linear Algebra Subprograms Technical (BLAST) Forum (2001) *Basic Linear Algebra Subprograms Technical (BLAST) Forum Standard* University of Tennessee, Knoxville, Tennessee <http://www.netlib.org/blas/blast-forum/blas-report.pdf>

## 5 Arguments

1: INORM – INTEGER *Input*

*On entry:* specifies the value to be returned. The integer codes shown below can be replaced by the equivalent named constants of the form NAG\_?\_NORM. These named constants are available via the **nag\_library** module and are also used in the example program for clarity.

INORM = 171 (NAG\_ONE\_NORM)  
The 1-norm.

INORM = 173 (NAG\_TWO\_NORM)  
The 2-norm of a row or column vector.

INORM = 174 (NAG\_FROBENIUS\_NORM)  
The Frobenius (or Euclidean) norm.

INORM = 175 (NAG\_INF\_NORM)  
The  $\infty$ -norm.

INORM = 177 (NAG\_MAX\_NORM)  
The value  $\max_{i,j} |a_{ij}|$  (not a norm).

*Constraints:*

INORM = 171, 173, 174, 175 or 177;  
if INORM = 173, M = 1 or N = 1.

- 2: M – INTEGER *Input*  
*On entry:*  $m$ , the number of rows of the matrix  $A$ . If  $M \leq 0$  on input, F16RBF returns 0.
- 3: N – INTEGER *Input*  
*On entry:*  $n$ , the number of columns of the matrix  $A$ . If  $N \leq 0$  on input, F16RBF returns 0.
- 4: KL – INTEGER *Input*  
*On entry:*  $k_l$ , the number of subdiagonals within the band of  $A$ . If  $KL \leq 0$  on input, F16RBF returns 0.
- 5: KU – INTEGER *Input*  
*On entry:*  $k_u$ , the number of superdiagonals within the band of  $A$ . If  $KU \leq 0$  on input, F16RBF returns 0.
- 6: AB(LDAB,\*) – REAL (KIND=nag\_wp) array *Input*  
**Note:** the second dimension of the array AB must be at least  $\max(1, N)$ .  
*On entry:* the  $m$  by  $n$  band matrix  $A$ .  
The matrix is stored in rows 1 to  $k_l + k_u + 1$ , more precisely, the element  $A_{ij}$  must be stored in  
$$AB(k_u + 1 + i - j, j) \quad \text{for } \max(1, j - k_u) \leq i \leq \min(m, j + k_l).$$
- 7: LDAB – INTEGER *Input*  
*On entry:* the first dimension of the array AB as declared in the (sub)program from which F16RBF is called.  
*Constraint:*  $LDAB \geq KL + KU + 1$ .

## 6 Error Indicators and Warnings

If any constraint on an input parameter is violated, an error message is printed and program execution is terminated.

## 7 Accuracy

The BLAS standard requires accurate implementations which avoid unnecessary over/underflow (see Section 2.7 of Basic Linear Algebra Subprograms Technical (BLAST) Forum (2001)).

## 8 Parallelism and Performance

F16RBF is not threaded in any implementation.

## 9 Further Comments

None.

## 10 Example

Calculates the various norms of a 6 by 4 banded matrix with two subdiagonals and one superdiagonal.

### 10.1 Program Text

```

Program f16rbfe

!      F16RBF Example Program Text

!      Mark 26 Release. NAG Copyright 2016.

!      .. Use Statements ..
Use nag_library, Only: f01zcf, f16rbf, nag_frobenius_norm, nag_inf_norm, &
                        nag_max_norm, nag_one_norm, nag_wp
!      .. Implicit None Statement ..
Implicit None
!      .. Parameters ..
Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
Real (Kind=nag_wp)         :: r_fro, r_inf, r_max, r_one
Integer                    :: i, ifail, j, kl, ku, lda, ldab, m, n
Character (1)              :: job
!      .. Local Arrays ..
Real (Kind=nag_wp), Allocatable :: a(:,,:), ab(:,:)
!      .. Intrinsic Procedures ..
Intrinsic                  :: max, min
!      .. Executable Statements ..
Write (nout,*) 'F16RBF Example Program Results'

!      Skip heading in data file
Read (nin,*)

      Read (nin,*) m, n, kl, ku
      lda = m
      ldab = kl + ku + 1
      Allocate (a(lda,n),ab(ldab,n))

!      Read A from data file into rectangular storage

      Do i = 1, m
         Read (nin,*)(a(i,j),j=max(1,i-kl),min(n,i+ku))
      End Do

!      Convert A to packed storage

      job = 'P'

      ifail = 0
      Call f01zcf(job,m,n,kl,ku,a,lda,ab,ldab,ifail)

      Write (nout,*)
      Write (nout,99999) 'Norms of banded matrix AB:'
      Write (nout,*)

      r_one = f16rbf(nag_one_norm,m,n,kl,ku,ab,ldab)
      Write (nout,99998) 'One norm          = ', r_one

      r_inf = f16rbf(nag_inf_norm,m,n,kl,ku,ab,ldab)
      Write (nout,99998) 'Infinity norm       = ', r_inf

      r_fro = f16rbf(nag_frobenius_norm,m,n,kl,ku,ab,ldab)
      Write (nout,99998) 'Frobenius norm      = ', r_fro

      r_max = f16rbf(nag_max_norm,m,n,kl,ku,ab,ldab)

```

```
      Write (nout,99998) 'Maximum norm      = ', r_max
99999 Format (1X,A)
99998 Format (1X,A,F9.4)
      End Program f16rbfe
```

## 10.2 Program Data

F16RBF Example Program Data

```
6 4 2 1                                : M, N, KL, KU
 1.0 1.0
 2.0 2.0 2.0
 3.0 3.0 3.0 3.0
   4.0 4.0 4.0
     5.0 5.0
       6.0                                : AB
```

## 10.3 Program Results

F16RBF Example Program Results

Norms of banded matrix AB:

```
One norm           = 18.0000
Infinity norm      = 12.0000
Frobenius norm     = 13.5647
Maximum norm       = 6.0000
```

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