NAG Library Routine Document

F08LEF (DGBBRD)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F08LEF (DGBBRD) reduces a real m by n band matrix to upper bidiagonal form.

2 Specification

```
SUBROUTINE F08LEF (VECT, M, N, NCC, KL, KU, AB, LDAB, D, E, Q, LDQ, PT,    &
                   LDPT, C, LDC, WORK, INFO)
INTEGER            M, N, NCC, KL, KU, LDAB, LDQ, LDPT, LDC, INFO
REAL (KIND=nag_wp) AB(LDAB,*), D(min(M,N)), E(min(M,N)-1), Q(LDQ,*),       &
                   PT(LDPT,*), C(LDC,*), WORK(2*max(M,N))
CHARACTER(1)       VECT
```

The routine may be called by its LAPACK name ***dgbbrd***.

3 Description

F08LEF (DGBBRD) reduces a real m by n band matrix to upper bidiagonal form B by an orthogonal transformation: $A = QBP^{\mathrm{T}}$. The orthogonal matrices Q and P^{T}, of order m and n respectively, are determined as a product of Givens rotation matrices, and may be formed explicitly by the routine if required. A matrix C may also be updated to give $\tilde{C} = Q^{\mathrm{T}}C$.

The routine uses a vectorizable form of the reduction.

4 References

None.

5 Arguments

1: VECT – CHARACTER(1) *Input*

On entry: indicates whether the matrices Q and/or P^{T} are generated.

VECT = 'N'
: Neither Q nor P^{T} is generated.

VECT = 'Q'
: Q is generated.

VECT = 'P'
: P^{T} is generated.

VECT = 'B'
: Both Q and P^{T} are generated.

Constraint: VECT = 'N', 'Q', 'P' or 'B'.

2: M – INTEGER *Input*

On entry: m, the number of rows of the matrix A.

Constraint: $\mathrm{M} \geq 0$.

3: N – INTEGER *Input*

On entry: n, the number of columns of the matrix A.

Constraint: $N \geq 0$.

4: NCC – INTEGER *Input*

On entry: n_C, the number of columns of the matrix C.

Constraint: $NCC \geq 0$.

5: KL – INTEGER *Input*

On entry: the number of subdiagonals, k_l, within the band of A.

Constraint: $KL \geq 0$.

6: KU – INTEGER *Input*

On entry: the number of superdiagonals, k_u, within the band of A.

Constraint: $KU \geq 0$.

7: AB(LDAB, $*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array AB must be at least $\max(1, N)$.

On entry: the original m by n band matrix A.

The matrix is stored in rows 1 to $k_l + k_u + 1$, more precisely, the element A_{ij} must be stored in

$$AB(k_u + 1 + i - j, j) \quad \text{for } \max(1, j - k_u) \leq i \leq \min(m, j + k_l).$$

On exit: AB is overwritten by values generated during the reduction.

8: LDAB – INTEGER *Input*

On entry: the first dimension of the array AB as declared in the (sub)program from which F08LEF (DGBBRD) is called.

Constraint: $LDAB \geq KL + KU + 1$.

9: D(min(M, N)) – REAL (KIND=nag_wp) array *Output*

On exit: the diagonal elements of the bidiagonal matrix B.

10: E(min(M, N) − 1) – REAL (KIND=nag_wp) array *Output*

On exit: the superdiagonal elements of the bidiagonal matrix B.

11: Q(LDQ, $*$) – REAL (KIND=nag_wp) array *Output*

Note: the second dimension of the array Q must be at least $\max(1, M)$ if VECT = 'Q' or 'B', and at least 1 otherwise.

On exit: if VECT = 'Q' or 'B', contains the m by m orthogonal matrix Q.

If VECT = 'N' or 'P', Q is not referenced.

12: LDQ – INTEGER *Input*

On entry: the first dimension of the array Q as declared in the (sub)program from which F08LEF (DGBBRD) is called.

Constraints:

if VECT = 'Q' or 'B', $LDQ \geq \max(1, M)$;
otherwise $LDQ \geq 1$.

13: PT(LDPT, $*$) – REAL (KIND=nag_wp) array *Output*

Note: the second dimension of the array PT must be at least $\max(1, \text{N})$ if VECT = 'P' or 'B', and at least 1 otherwise.

On exit: the n by n orthogonal matrix P^{T}, if VECT = 'P' or 'B'. If VECT = 'N' or 'Q', PT is not referenced.

14: LDPT – INTEGER *Input*

On entry: the first dimension of the array PT as declared in the (sub)program from which F08LEF (DGBBRD) is called.

Constraints:

if VECT = 'P' or 'B', LDPT $\geq \max(1, \text{N})$;
otherwise LDPT ≥ 1.

15: C(LDC, $*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array C must be at least $\max(1, \text{NCC})$.

On entry: an m by n_C matrix C.

On exit: C is overwritten by $Q^{\mathrm{T}}C$. If NCC $= 0$, C is not referenced.

16: LDC – INTEGER *Input*

On entry: the first dimension of the array C as declared in the (sub)program from which F08LEF (DGBBRD) is called.

Constraints:

if NCC > 0, LDC $\geq \max(1, \text{M})$;
if NCC $= 0$, LDC ≥ 1.

17: WORK($2 \times \max(\text{M}, \text{N})$) – REAL (KIND=nag_wp) array *Workspace*

18: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed bidiagonal form B satisfies $QBP^{\mathrm{T}} = A + E$, where

$$\|E\|_2 \leq c(n)\epsilon\|A\|_2,$$

$c(n)$ is a modestly increasing function of n, and ϵ is the ***machine precision***.

The elements of B themselves may be sensitive to small perturbations in A or to rounding errors in the computation, but this does not affect the stability of the singular values and vectors.

The computed matrix Q differs from an exactly orthogonal matrix by a matrix F such that

$$\|F\|_2 = O(\epsilon).$$

A similar statement holds for the computed matrix P^{T}.

8 Parallelism and Performance

F08LEF (DGBBRD) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The total number of real floating-point operations is approximately the sum of:

$6n^2k$, if VECT = 'N' and NCC = 0, and

$3n^2n_C(k-1)/k$, if C is updated, and

$3n^3(k-1)/k$, if either Q or P^{T} is generated (double this if both),

where $k = k_l + k_u$, assuming $n \gg k$. For this section we assume that $m = n$.

The complex analogue of this routine is F08LSF (ZGBBRD).

10 Example

This example reduces the matrix A to upper bidiagonal form, where

$$A = \begin{pmatrix} -0.57 & -1.28 & 0.00 & 0.00 \\ -1.93 & 1.08 & -0.31 & 0.00 \\ 2.30 & 0.24 & 0.40 & -0.35 \\ 0.00 & 0.64 & -0.66 & 0.08 \\ 0.00 & 0.00 & 0.15 & -2.13 \\ -0.00 & 0.00 & 0.00 & 0.50 \end{pmatrix}.$$

10.1 Program Text

```
    Program f08lefe

!     F08LEF Example Program Text

!     Mark 26 Release. NAG Copyright 2016.

!     .. Use Statements ..
      Use nag_library, Only: dgbbrd, nag_wp
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
      Character (1), Parameter         :: vect = 'B'
!     .. Local Scalars ..
      Integer                          :: i, info, j, kl, ku, ldab, ldb, ldc,  &
                                          ldpt, ldq, m, n, ncc
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable  :: ab(:,:), b(:,:), c(:,:), d(:), e(:), &
                                          pt(:,:), q(:,:), work(:)
!     .. Intrinsic Procedures ..
      Intrinsic                        :: abs, max, min
!     .. Executable Statements ..
      Write (nout,*) 'F08LEF Example Program Results'
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) m, n, kl, ku, ncc
      ldab = kl + ku + 1
      ldb = m
      ldc = m
      ldpt = n
```

```
      ldq = m
      Allocate (ab(ldab,n),b(ldb,n),c(m,ncc),d(n),e(n-1),pt(ldpt,n),q(ldq,m),  &
        work(2*m+2*n))

!     Read A from data file

      Read (nin,*)((ab(ku+1+i-j,j),j=max(i-kl,1),min(i+ku,n)),i=1,m)

!     Reduce A to upper bidiagonal form
!     The NAG name equivalent of dgbbrd is f08lef
      Call dgbbrd(vect,m,n,ncc,kl,ku,ab,ldab,d,e,q,ldq,pt,ldpt,c,ldc,work,     &
        info)

!     Print the absolute values of bidiagonal vectors d and e.
!     Any of these can differ by a sign change by combinations of sign
!     changes in columns of Q and P (rows of PT).
      Write (nout,*)
      Write (nout,*) 'Diagonal D:'
      Write (nout,99999) abs(d(1:n))
      Write (nout,*)
      Write (nout,*) 'Off-diagonal E:'
      Write (nout,99999) abs(e(1:n-1))
99999 Format (1X,4(3X,F11.4))

    End Program f08lefe
```

10.2 Program Data

```
F08LEF Example Program Data
  6  4  2  1  0               :Values of M, N, KL, KU and NCC
 -0.57  -1.28
 -1.93   1.08  -0.31
  2.30   0.24   0.40  -0.35
         0.64  -0.66   0.08
                0.15  -2.13
                       0.50   :End of matrix A
```

10.3 Program Results

```
 F08LEF Example Program Results

 Diagonal D:
        3.0561        1.5259        0.9690        1.5685

 Off-diagonal E:
        0.6206        1.2353        1.1240
```