

# NAG Library Routine Document

## F07UEF (DTPTRS)

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

### 1 Purpose

F07UEF (DTPTRS) solves a real triangular system of linear equations with multiple right-hand sides,  $AX = B$  or  $A^T X = B$ , using packed storage.

### 2 Specification

```
SUBROUTINE F07UEF (UPLO, TRANS, DIAG, N, NRHS, AP, B, LDB, INFO)
  INTEGER          N, NRHS, LDB, INFO
  REAL (KIND=nag_wp) AP(*), B(LDB,*)
  CHARACTER(1)    UPLO, TRANS, DIAG
```

The routine may be called by its LAPACK name *dtptrs*.

### 3 Description

F07UEF (DTPTRS) solves a real triangular system of linear equations  $AX = B$  or  $A^T X = B$ , using packed storage.

### 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J (1989) The accuracy of solutions to triangular systems *SIAM J. Numer. Anal.* **26** 1252–1265

### 5 Arguments

- 1: UPLO – CHARACTER(1) *Input*  
*On entry:* specifies whether  $A$  is upper or lower triangular.  
 UPLO = 'U'  
      $A$  is upper triangular.  
 UPLO = 'L'  
      $A$  is lower triangular.  
*Constraint:* UPLO = 'U' or 'L'.
- 2: TRANS – CHARACTER(1) *Input*  
*On entry:* indicates the form of the equations.  
 TRANS = 'N'  
     The equations are of the form  $AX = B$ .  
 TRANS = 'T' or 'C'  
     The equations are of the form  $A^T X = B$ .  
*Constraint:* TRANS = 'N', 'T' or 'C'.

- 3:    DIAG – CHARACTER(1) *Input*  
*On entry:* indicates whether  $A$  is a nonunit or unit triangular matrix.  
 DIAG = 'N'  
        $A$  is a nonunit triangular matrix.  
 DIAG = 'U'  
        $A$  is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.  
*Constraint:* DIAG = 'N' or 'U'.
- 4:    N – INTEGER *Input*  
*On entry:*  $n$ , the order of the matrix  $A$ .  
*Constraint:*  $N \geq 0$ .
- 5:    NRHS – INTEGER *Input*  
*On entry:*  $r$ , the number of right-hand sides.  
*Constraint:* NRHS  $\geq 0$ .
- 6:    AP(\*) – REAL (KIND=nag\_wp) array *Input*  
**Note:** the dimension of the array AP must be at least  $\max(1, N \times (N + 1)/2)$ .  
*On entry:* the  $n$  by  $n$  triangular matrix  $A$ , packed by columns.  
 More precisely,  
       if UPLO = 'U', the upper triangle of  $A$  must be stored with element  $A_{ij}$  in  
       AP( $i + j(j - 1)/2$ ) for  $i \leq j$ ;  
       if UPLO = 'L', the lower triangle of  $A$  must be stored with element  $A_{ij}$  in  
       AP( $i + (2n - j)(j - 1)/2$ ) for  $i \geq j$ .  
 If DIAG = 'U', the diagonal elements of  $A$  are assumed to be 1, and are not referenced; the same storage scheme is used whether DIAG = 'N' or 'U'.
- 7:    B(LDB, \*) – REAL (KIND=nag\_wp) array *Input/Output*  
**Note:** the second dimension of the array B must be at least  $\max(1, \text{NRHS})$ .  
*On entry:* the  $n$  by  $r$  right-hand side matrix  $B$ .  
*On exit:* the  $n$  by  $r$  solution matrix  $X$ .
- 8:    LDB – INTEGER *Input*  
*On entry:* the first dimension of the array B as declared in the (sub)program from which F07UEF (DTPTRS) is called.  
*Constraint:* LDB  $\geq \max(1, N)$ .
- 9:    INFO – INTEGER *Output*  
*On exit:* INFO = 0 unless the routine detects an error (see Section 6).

## 6 Error Indicators and Warnings

INFO < 0

If INFO =  $-i$ , argument  $i$  had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

Element  $\langle value \rangle$  of the diagonal is exactly zero.  $A$  is singular and the solution has not been computed.

## 7 Accuracy

The solutions of triangular systems of equations are usually computed to high accuracy. See Higham (1989).

For each right-hand side vector  $b$ , the computed solution  $x$  is the exact solution of a perturbed system of equations  $(A + E)x = b$ , where

$$|E| \leq c(n)\epsilon|A|,$$

$c(n)$  is a modest linear function of  $n$ , and  $\epsilon$  is the *machine precision*.

If  $\hat{x}$  is the true solution, then the computed solution  $x$  satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_{\infty}}{\|x\|_{\infty}} \leq c(n) \text{cond}(A, x)\epsilon, \quad \text{provided} \quad c(n) \text{cond}(A, x)\epsilon < 1,$$

where  $\text{cond}(A, x) = \| |A^{-1}| |A| |x| \|_{\infty} / \|x\|_{\infty}$ .

Note that  $\text{cond}(A, x) \leq \text{cond}(A) = \| |A^{-1}| |A| \|_{\infty} \leq \kappa_{\infty}(A)$ ;  $\text{cond}(A, x)$  can be much smaller than  $\text{cond}(A)$  and it is also possible for  $\text{cond}(A^T)$  to be much larger (or smaller) than  $\text{cond}(A)$ .

Forward and backward error bounds can be computed by calling F07UHF (DTPRFS), and an estimate for  $\kappa_{\infty}(A)$  can be obtained by calling F07UGF (DTPCON) with NORM = 'I'.

## 8 Parallelism and Performance

F07UEF (DTPTRS) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

F07UEF (DTPTRS) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The total number of floating-point operations is approximately  $n^2r$ .

The complex analogue of this routine is F07USF (ZTPTRS).

## 10 Example

This example solves the system of equations  $AX = B$ , where

$$A = \begin{pmatrix} 4.30 & 0.00 & 0.00 & 0.00 \\ -3.96 & -4.87 & 0.00 & 0.00 \\ 0.40 & 0.31 & -8.02 & 0.00 \\ -0.27 & 0.07 & -5.95 & 0.12 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -12.90 & -21.50 \\ 16.75 & 14.93 \\ -17.55 & 6.33 \\ -11.04 & 8.09 \end{pmatrix},$$

using packed storage for  $A$ .

## 10.1 Program Text

```

Program f07uefe

!      F07UEF Example Program Text

!      Mark 26 Release. NAG Copyright 2016.

!      .. Use Statements ..
Use nag_library, Only: dtptrs, nag_wp, x04caf
!      .. Implicit None Statement ..
Implicit None
!      .. Parameters ..
Integer, Parameter           :: nin = 5, nout = 6
Character (1), Parameter    :: diag = 'N', trans = 'N'
!      .. Local Scalars ..
Integer                      :: i, ifail, info, j, ldb, n, nrhs
Character (1)                :: uplo
!      .. Local Arrays ..
Real (Kind=nag_wp), Allocatable :: ap(:), b(:, :)
!      .. Executable Statements ..
Write (nout,*) 'F07UEF Example Program Results'
!      Skip heading in data file
Read (nin,*)
Read (nin,*) n, nrhs
ldb = n
Allocate (ap(n*(n+1)/2),b(ldb,nrhs))

!      Read A and B from data file

Read (nin,*) uplo
If (uplo=='U') Then
  Read (nin,*)((ap(i+j*(j-1)/2),j=i,n),i=1,n)
Else If (uplo=='L') Then
  Read (nin,*)((ap(i+(2*n-j)*(j-1)/2),j=1,i),i=1,n)
End If
Read (nin,*)(b(i,1:nrhs),i=1,n)

!      Compute solution
!      The NAG name equivalent of dtptrs is f07uef
Call dtptrs(uplo,trans,diag,n,nrhs,ap,b,ldb,info)

!      Print solution

Write (nout,*)
Flush (nout)
If (info==0) Then

!      ifail: behaviour on error exit
!      =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
  ifail = 0
  Call x04caf('General',' ',n,nrhs,b,ldb,'Solution(s)',ifail)

Else
  Write (nout,*) 'A is singular'
End If

End Program f07uefe

```

## 10.2 Program Data

```

F07UEF Example Program Data
  4  2           :Values of N and NRHS
  'L'           :Value of UPL0
  4.30
 -3.96  -4.87
  0.40   0.31  -8.02

```

```
-0.27  0.07 -5.95  0.12  :End of matrix A
-12.90 -21.50
 16.75  14.93
-17.55  6.33
-11.04  8.09              :End of matrix B
```

### 10.3 Program Results

F07UEF Example Program Results

```
Solution(s)
           1           2
1      -3.0000    -5.0000
2      -1.0000     1.0000
3       2.0000    -1.0000
4       1.0000     6.0000
```

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