

NAG Library Routine Document

F07JUF (ZPTCON)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F07JUF (ZPTCON) computes the reciprocal condition number of a complex n by n Hermitian positive definite tridiagonal matrix A , using the LDL^H factorization returned by F07JRF (ZPTTRF).

2 Specification

```
SUBROUTINE F07JUF (N, D, E, ANORM, RCOND, RWORK, INFO)
  INTEGER          N, INFO
  REAL (KIND=nag_wp) D(*), ANORM, RCOND, RWORK(N)
  COMPLEX (KIND=nag_wp) E(*)
```

The routine may be called by its LAPACK name *zptcon*.

3 Description

F07JUF (ZPTCON) should be preceded by a call to F07JRF (ZPTTRF), which computes a modified Cholesky factorization of the matrix A as

$$A = LDL^H,$$

where L is a unit lower bidiagonal matrix and D is a diagonal matrix, with positive diagonal elements. F07JUF (ZPTCON) then utilizes the factorization to compute $\|A^{-1}\|_1$ by a direct method, from which the reciprocal of the condition number of A , $1/\kappa(A)$ is computed as

$$1/\kappa_1(A) = 1/(\|A\|_1\|A^{-1}\|_1).$$

$1/\kappa(A)$ is returned, rather than $\kappa(A)$, since when A is singular $\kappa(A)$ is infinite.

4 References

Higham N J (2002) *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

5 Arguments

1: N – INTEGER *Input*

On entry: n , the order of the matrix A .

Constraint: $N \geq 0$.

2: D(*) – REAL (KIND=nag_wp) array *Input*

Note: the dimension of the array D must be at least $\max(1, N)$.

On entry: must contain the n diagonal elements of the diagonal matrix D from the LDL^H factorization of A .

- 3: E(*) – COMPLEX (KIND=nag_wp) array Input
Note: the dimension of the array E must be at least $\max(1, N - 1)$.
On entry: must contain the $(n - 1)$ subdiagonal elements of the unit lower bidiagonal matrix L . (E can also be regarded as the superdiagonal of the unit upper bidiagonal matrix U from the $U^H D U$ factorization of A .)
- 4: ANORM – REAL (KIND=nag_wp) Input
On entry: the 1-norm of the **original** matrix A , which may be computed by calling F06UPF with its argument NORM = '1'. ANORM must be computed either **before** calling F07JRF (ZPTTRF) or else from a **copy** of the original matrix A .
Constraint: ANORM \geq 0.0.
- 5: RCOND – REAL (KIND=nag_wp) Output
On exit: the reciprocal condition number, $1/\kappa_1(A) = 1/(\|A\|_1 \|A^{-1}\|_1)$.
- 6: RWORK(N) – REAL (KIND=nag_wp) array Workspace
- 7: INFO – INTEGER Output
On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO = $-i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed condition number will be the exact condition number for a closely neighbouring matrix.

8 Parallelism and Performance

F07JUF (ZPTCON) is not threaded in any implementation.

9 Further Comments

The condition number estimation requires $O(n)$ floating-point operations.

See Section 15.6 of Higham (2002) for further details on computing the condition number of tridiagonal matrices.

The real analogue of this routine is F07JGF (DPTCON).

10 Example

This example computes the condition number of the Hermitian positive definite tridiagonal matrix A given by

$$A = \begin{pmatrix} 16.0 & 16.0 - 16.0i & 0 & 0 \\ 16.0 + 16.0i & 41.0 & 18.0 + 9.0i & 0 \\ 0 & 18.0 - 9.0i & 46.0 & 1.0 + 4.0i \\ 0 & 0 & 1.0 - 4.0i & 21.0 \end{pmatrix}.$$

10.1 Program Text

```

Program f07jufe

!      F07JUF Example Program Text

!      Mark 26 Release. NAG Copyright 2016.

!      .. Use Statements ..
Use nag_library, Only: f06upf, nag_wp, x02ajf, zptcon, zpttrf
!      .. Implicit None Statement ..
Implicit None
!      .. Parameters ..
Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
Real (Kind=nag_wp)         :: anorm, rcond
Integer                     :: info, n
!      .. Local Arrays ..
Complex (Kind=nag_wp), Allocatable :: e(:)
Real (Kind=nag_wp), Allocatable  :: d(:), rwork(:)
!      .. Executable Statements ..
Write (nout,*) 'F07JUF Example Program Results'
Write (nout,*)
!      Skip heading in data file
Read (nin,*)
Read (nin,*) n

Allocate (e(n-1),d(n),rwork(n))

!      Read the lower bidiagonal part of the tridiagonal matrix A from
!      data file

Read (nin,*) d(1:n)
Read (nin,*) e(1:n-1)

!      Compute the 1-norm of A
anorm = f06upf('1-norm',n,d,e)

!      Factorize the tridiagonal matrix A
!      The NAG name equivalent of zpttrf is f07jrf
Call zpttrf(n,d,e,info)

If (info==0) Then

!      Estimate the condition number of A
!      The NAG name equivalent of zptcon is f07juf
Call zptcon(n,d,e,anorm,rcond,rwork,info)

!      Print the estimated condition number

If (rcond>=x02ajf()) Then
  Write (nout,99999) 'Estimate of condition number = ',          &
    1.0E0_nag_wp/rcond
Else
  Write (nout,99999) 'A is singular to working precision. RCOND = ', &
    rcond
End If

Else
  Write (nout,99998) 'The leading minor of order ', info,          &
    ' is not positive definite'
End If

99999 Format (1X,A,1P,E10.2)
99998 Format (1X,A,I3,A)
End Program f07jufe

```

10.2 Program Data

F07JUF Example Program Data

```
4
16.0      41.0      46.0      21.0      :Value of N
( 16.0, 16.0) ( 18.0, -9.0) ( 1.0, -4.0) :End of diagonal D
                                           :End of sub-diagonal E
```

10.3 Program Results

F07JUF Example Program Results

Estimate of condition number = 9.21E+03
