NAG Library Routine Document

F07JDF (DPTTRF)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07JDF (DPTTRF) computes the modified Cholesky factorization of a real n by n symmetric positive definite tridiagonal matrix A.

2 Specification

```
SUBROUTINE F07JDF (N, D, E, INFO)
INTEGER                 N, INFO
REAL (KIND=nag_wp) D(*), E(*)
```

The routine may be called by its LAPACK name ***dpttrf***.

3 Description

F07JDF (DPTTRF) factorizes the matrix A as

$$A = LDL^{\mathrm{T}},$$

where L is a unit lower bidiagonal matrix and D is a diagonal matrix with positive diagonal elements. The factorization may also be regarded as having the form $U^{\mathrm{T}}DU$, where U is a unit upper bidiagonal matrix.

4 References

None.

5 Arguments

1: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

2: D($*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the dimension of the array D must be at least $\max(1, \mathrm{N})$.

On entry: must contain the n diagonal elements of the matrix A.

On exit: is overwritten by the n diagonal elements of the diagonal matrix D from the LDL^{T} factorization of A.

3: E($*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the dimension of the array E must be at least $\max(1, \mathrm{N} - 1)$.

On entry: must contain the $(n-1)$ subdiagonal elements of the matrix A.

On exit: is overwritten by the $(n-1)$ subdiagonal elements of the lower bidiagonal matrix L. (E can also be regarded as containing the $(n-1)$ superdiagonal elements of the upper bidiagonal matrix U.)

4: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0 and INFO $<$ N

The leading minor of order $\langle value \rangle$ is not positive definite, the factorization could not be completed.

INFO > 0 and INFO $=$ N

The leading minor of order n is not positive definite, the factorization was completed, but D(N) ≤ 0.

7 Accuracy

The computed factorization satisfies an equation of the form

$$A + E = LDL^{\mathrm{T}},$$

where

$$\|E\|_\infty = O(\epsilon)\|A\|_\infty$$

and ϵ is the ***machine precision***.

Following the use of this routine, F07JEF (DPTTRS) can be used to solve systems of equations $AX = B$, and F07JGF (DPTCON) can be used to estimate the condition number of A.

8 Parallelism and Performance

F07JDF (DPTTRF) is not threaded in any implementation.

9 Further Comments

The total number of floating-point operations required to factorize the matrix A is proportional to n.

The complex analogue of this routine is F07JRF (ZPTTRF).

10 Example

This example factorizes the symmetric positive definite tridiagonal matrix A given by

$$A = \begin{pmatrix} 4.0 & -2.0 & 0 & 0 & 0 \\ -2.0 & 10.0 & -6.0 & 0 & 0 \\ 0 & -6.0 & 29.0 & 15.0 & 0 \\ 0 & 0 & 15.0 & 25.0 & 8.0 \\ 0 & 0 & 0 & 8.0 & 5.0 \end{pmatrix}.$$

10.1 Program Text

```
    Program f07jdfe

!     F07JDF Example Program Text

!     Mark 26 Release. NAG Copyright 2016.

!     .. Use Statements ..
      Use nag_library, Only: dpttrf, nag_wp
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                          :: info, n
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable  :: d(:), e(:)
!     .. Executable Statements ..
      Write (nout,*) 'F07JDF Example Program Results'
      Write (nout,*)
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n

      Allocate (d(n),e(n-1))

!     Read the lower bidiagonal part of the tridiagonal matrix A from
!     data file

      Read (nin,*) d(1:n)
      Read (nin,*) e(1:n-1)

!     Factorize the tridiagonal matrix A

!     The NAG name equivalent of dpttrf is f07jdf
      Call dpttrf(n,d,e,info)

      If (info>0) Then
        Write (nout,99999) 'The leading minor of order ', info,                &
          ' is not positive definite'
      End If

!     Print details of the factorization

      Write (nout,*) 'Details of factorization'
      Write (nout,*)
      Write (nout,*) ' The diagonal elements of D'
      Write (nout,99998) d(1:n)
      Write (nout,*)
      Write (nout,*) ' Subdiagonal elements of the Cholesky factor L'
      Write (nout,99998) e(1:n-1)

99999 Format (1X,A,I3,A)
99998 Format (1X,8F9.4)
    End Program f07jdfe
```

10.2 Program Data

```
F07JDF Example Program Data
  5                          :Value of N
  4.0  10.0  29.0  25.0   5.0 :End of diagonal D
 -2.0  -6.0  15.0   8.0       :End of sub-diagonal E
```

10.3 Program Results

```
 F07JDF Example Program Results

 Details of factorization

  The diagonal elements of D
    4.0000   9.0000  25.0000  16.0000   1.0000

  Subdiagonal elements of the Cholesky factor L
   -0.5000  -0.6667   0.6000   0.5000
```