

NAG Library Routine Document

F07GNF (ZPPSV)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F07GNF (ZPPSV) computes the solution to a complex system of linear equations

$$AX = B,$$

where A is an n by n Hermitian positive definite matrix stored in packed format and X and B are n by r matrices.

2 Specification

SUBROUTINE F07GNF (UPLO, N, NRHS, AP, B, LDB, INFO)

INTEGER N, NRHS, LDB, INFO

COMPLEX (KIND=nag_wp) AP(*), B(LDB,*)

CHARACTER(1) UPLO

The routine may be called by its LAPACK name *zppsv*.

3 Description

F07GNF (ZPPSV) uses the Cholesky decomposition to factor A as $A = U^H U$ if UPLO = 'U' or $A = L L^H$ if UPLO = 'L', where U is an upper triangular matrix and L is a lower triangular matrix. The factored form of A is then used to solve the system of equations $AX = B$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Arguments

- 1: UPLO – CHARACTER(1) *Input*
On entry: if UPLO = 'U', the upper triangle of A is stored.
 If UPLO = 'L', the lower triangle of A is stored.
Constraint: UPLO = 'U' or 'L'.
- 2: N – INTEGER *Input*
On entry: n , the number of linear equations, i.e., the order of the matrix A .
Constraint: $N \geq 0$.
- 3: NRHS – INTEGER *Input*
On entry: r , the number of right-hand sides, i.e., the number of columns of the matrix B .
Constraint: NRHS ≥ 0 .

4: AP(*) – COMPLEX (KIND=nag_wp) array Input/Output

Note: the dimension of the array AP must be at least $\max(1, N \times (N + 1)/2)$.

On entry: the n by n Hermitian matrix A , packed by columns.

More precisely,

if UPLO = 'U', the upper triangle of A must be stored with element A_{ij} in $AP(i + j(j - 1)/2)$ for $i \leq j$;

if UPLO = 'L', the lower triangle of A must be stored with element A_{ij} in $AP(i + (2n - j)(j - 1)/2)$ for $i \geq j$.

On exit: if INFO = 0, the factor U or L from the Cholesky factorization $A = U^H U$ or $A = LL^H$, in the same storage format as A .

5: B(LDB, *) – COMPLEX (KIND=nag_wp) array Input/Output

Note: the second dimension of the array B must be at least $\max(1, NRHS)$.

Note: to solve the equations $Ax = b$, where b is a single right-hand side, B may be supplied as a one-dimensional array with length $LDB = \max(1, N)$.

On entry: the n by r right-hand side matrix B .

On exit: if INFO = 0, the n by r solution matrix X .

6: LDB – INTEGER Input

On entry: the first dimension of the array B as declared in the (sub)program from which F07GNF (ZPPSV) is called.

Constraint: $LDB \geq \max(1, N)$.

7: INFO – INTEGER Output

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO = $-i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

The leading minor of order $\langle value \rangle$ of A is not positive definite, so the factorization could not be completed, and the solution has not been computed.

7 Accuracy

The computed solution for a single right-hand side, \hat{x} , satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the *machine precision*. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A) \frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

F07GPF (ZPPSVX) is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, F04CEF solves $Ax = b$ and returns a forward error bound and condition estimate. F04CEF calls F07GNF (ZPPSV) to solve the equations.

8 Parallelism and Performance

F07GNF (ZPPSV) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

F07GNF (ZPPSV) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The total number of floating-point operations is approximately $\frac{4}{3}n^3 + 8n^2r$, where r is the number of right-hand sides.

The real analogue of this routine is F07GAF (DPPSV).

10 Example

This example solves the equations

$$Ax = b,$$

where A is the Hermitian positive definite matrix

$$A = \begin{pmatrix} 3.23 & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 \end{pmatrix}$$

and

$$b = \begin{pmatrix} 3.93 - 6.14i \\ 6.17 + 9.42i \\ -7.17 - 21.83i \\ 1.99 - 14.38i \end{pmatrix}.$$

Details of the Cholesky factorization of A are also output.

10.1 Program Text

```

Program f07gnfe

!      F07GNF Example Program Text

!      Mark 26 Release. NAG Copyright 2016.

!      .. Use Statements ..
!      Use nag_library, Only: nag_wp, x04ddf, zppsv
!      .. Implicit None Statement ..
!      Implicit None
!      .. Parameters ..
!      Integer, Parameter          :: nin = 5, nout = 6
!      Character (1), Parameter    :: uplo = 'U'
!      .. Local Scalars ..

```

```

Integer                                :: i, ifail, info, j, n
! .. Local Arrays ..
Complex (Kind=nag_wp), Allocatable :: ap(:), b(:)
Character (1)                          :: clabs(1), rlabs(1)
! .. Executable Statements ..
Write (nout,*) 'F07GNF Example Program Results'
Write (nout,*)
! Skip heading in data file
Read (nin,*)
Read (nin,*) n

Allocate (ap((n*(n+1))/2),b(n))

! Read the upper or lower triangular part of the matrix A from
! data file

If (uplo=='U') Then
  Read (nin,*)((ap(i+(j*(j-1))/2),j=i,n),i=1,n)
Else If (uplo=='L') Then
  Read (nin,*)((ap(i+((2*n-j)*(j-1))/2),j=1,i),i=1,n)
End If

! Read b from data file

Read (nin,*) b(1:n)

! Solve the equations Ax = b for x
! The NAG name equivalent of zppsv is f07gnf
Call zppsv(uplo,n,1,ap,b,n,info)

If (info==0) Then

!   Print solution

  Write (nout,*) 'Solution'
  Write (nout,99999) b(1:n)

!   Print details of factorization

  Write (nout,*)
  Flush (nout)

!   ifail: behaviour on error exit
!   =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
  ifail = 0
  Call x04ddf(uplo,'Non-unit diagonal',n,ap,'Bracketed','F7.4',      &
    'Cholesky factor','Integer',rlabs,'Integer',clabs,80,0,ifail)

  Else
    Write (nout,99998) 'The leading minor of order ', info,      &
      ' is not positive definite'
  End If

99999 Format ((3X,4(' (',F7.4,',',F7.4,')',:)))
99998 Format (1X,A,I3,A)
End Program f07gnfe

```

10.2 Program Data

F07GNF Example Program Data

```

4                                     :Value of N
( 3.23,  0.00) ( 1.51, -1.92) ( 1.90,  0.84) ( 0.42,  2.50)
              ( 3.58,  0.00) (-0.23,  1.11) (-1.18,  1.37)
              ( 4.09,  0.00) ( 2.33, -0.14)
              ( 4.29,  0.00) :End of matrix A
( 3.93, -6.14) ( 6.17,  9.42) (-7.17,-21.83) ( 1.99,-14.38) :End of vector b

```

10.3 Program Results

F07GNF Example Program Results

Solution

(1.0000,-1.0000) (-0.0000, 3.0000) (-4.0000,-5.0000) (2.0000, 1.0000)

Cholesky factor

	1	2	3	4
1	(1.7972, 0.0000)	(0.8402,-1.0683)	(1.0572, 0.4674)	(0.2337, 1.3910)
2		(1.3164, 0.0000)	(-0.4702,-0.3131)	(0.0834,-0.0368)
3			(1.5604, 0.0000)	(0.9360,-0.9900)
4				(0.6603, 0.0000)
