

NAG Library Routine Document

F07FTF (ZPOEQU)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F07FTF (ZPOEQU) computes a diagonal scaling matrix S intended to equilibrate a complex n by n Hermitian positive definite matrix A and reduce its condition number.

2 Specification

```
SUBROUTINE F07FTF (N, A, LDA, S, SCOND, AMAX, INFO)
INTEGER          N, LDA, INFO
REAL (KIND=nag_wp) S(N), SCOND, AMAX
COMPLEX (KIND=nag_wp) A(LDA,*)
```

The routine may be called by its LAPACK name *zpoequ*.

3 Description

F07FTF (ZPOEQU) computes a diagonal scaling matrix S chosen so that

$$s_j = 1/\sqrt{a_{jj}}.$$

This means that the matrix B given by

$$B = SAS,$$

has diagonal elements equal to unity. This in turn means that the condition number of B , $\kappa_2(B)$, is within a factor n of the matrix of smallest possible condition number over all possible choices of diagonal scalings (see Corollary 7.6 of Higham (2002)).

4 References

Higham N J (2002) *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

5 Arguments

- 1: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.
- 2: A(LDA,*) – COMPLEX (KIND=nag_wp) array *Input*
Note: the second dimension of the array A must be at least $\max(1, N)$.
On entry: the matrix A whose scaling factors are to be computed. Only the diagonal elements of the array A are referenced.
- 3: LDA – INTEGER *Input*
On entry: the first dimension of the array A as declared in the (sub)program from which F07FTF (ZPOEQU) is called.
Constraint: $LDA \geq \max(1, N)$.

- 4: S(N) – REAL (KIND=nag_wp) array Output
On exit: if INFO = 0, S contains the diagonal elements of the scaling matrix S .
- 5: SCOND – REAL (KIND=nag_wp) Output
On exit: if INFO = 0, SCOND contains the ratio of the smallest value of S to the largest value of S. If SCOND ≥ 0.1 and AMAX is neither too large nor too small, it is not worth scaling by S.
- 6: AMAX – REAL (KIND=nag_wp) Output
On exit: $\max |a_{ij}|$. If AMAX is very close to overflow or underflow, the matrix A should be scaled.
- 7: INFO – INTEGER Output
On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO = $-i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

The $\langle value \rangle$ th diagonal element of A is not positive (and hence A cannot be positive definite).

7 Accuracy

The computed scale factors will be close to the exact scale factors.

8 Parallelism and Performance

F07FTF (ZPOEQU) is not threaded in any implementation.

9 Further Comments

The real analogue of this routine is F07FFF (DPOEQU).

10 Example

This example equilibrates the Hermitian positive definite matrix A given by

$$A = \begin{pmatrix} 3.23 & 1.51 - 1.92i & (1.90 + 0.84i) \times 10^5 & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 & (-0.23 + 1.11i) \times 10^5 & -1.18 + 1.37i \\ (1.90 - 0.84i) \times 10^5 & (-0.23 - 1.11i) \times 10^5 & 4.09 \times 10^{10} & (2.33 - 0.14i) \times 10^5 \\ 0.42 - 2.50i & -1.18 - 1.37i & (2.33 + 0.14i) \times 10^5 & 4.29 \end{pmatrix}.$$

Details of the scaling factors and the scaled matrix are output.

10.1 Program Text

```

Program f07ftfe
!
!   F07FTF Example Program Text
!
!   Mark 26 Release. NAG Copyright 2016.
!
!   .. Use Statements ..

```

```

Use nag_library, Only: f06kcf, nag_wp, x02ajf, x02amf, x02bhf, x04dbf, &
                        zdscal, zpoequ
!
! .. Implicit None Statement ..
Implicit None
!
! .. Parameters ..
Real (Kind=nag_wp), Parameter      :: one = 1.0_nag_wp
Real (Kind=nag_wp), Parameter      :: thresh = 0.1_nag_wp
Integer, Parameter                 :: nin = 5, nout = 6
!
! .. Local Scalars ..
Real (Kind=nag_wp)                 :: amax, big, scond, small
Integer                             :: i, ifail, info, j, lda, n
!
! .. Local Arrays ..
Complex (Kind=nag_wp), Allocatable :: a(:, :)
Real (Kind=nag_wp), Allocatable    :: s(:)
Character (1)                       :: clabs(1), rlabs(1)
!
! .. Intrinsic Procedures ..
Intrinsic                           :: real
!
! .. Executable Statements ..
Write (nout,*) 'F07FTF Example Program Results'
Write (nout,*)
Flush (nout)
!
! Skip heading in data file
Read (nin,*)
Read (nin,*) n
lda = n
Allocate (a(lda,n),s(n))

!
! Read the upper triangular part of the matrix A from data file
Read (nin,*)(a(i,i:n),i=1,n)

!
! Print the matrix A

!
! ifail: behaviour on error exit
!           =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
ifail = 0
Call x04dbf('Upper', 'Non-unit', n, n, a, lda, 'Bracketed', '1P, E10.2', &
           'Matrix A', 'Integer', rlabs, 'Integer', clabs, 80, 0, ifail)

Write (nout,*)

!
! Compute diagonal scaling factors

!
! The NAG name equivalent of zpoequ is f07ftf
Call zpoequ(n, a, lda, s, scond, amax, info)

If (info>0) Then
  Write (nout,99999) 'Diagonal element', info, ' of A is non positive'
Else

!
!   Print SCOND, AMAX and the scale factors

  Write (nout,99998) 'SCOND =', scond, ', AMAX =', amax
  Write (nout,*)
  Write (nout,*) 'Diagonal scaling factors'
  Write (nout,99997) s(1:n)
  Write (nout,*)
  Flush (nout)

!
!   Compute values close to underflow and overflow

  small = x02amf()/(x02ajf()*real(x02bhf(),kind=nag_wp))
  big = one/small
  If ((scond<thresh) .Or. (amax<small) .Or. (amax>big)) Then

!
!     Scale A
!
!     The NAG name equivalent of zdscal is f06jdf
  Do j = 1, n
    Call zdscal(j,s(j),a(1,j),1)
    Call f06kcf(j,s,1,a(1,j),1)
  End Do

```

```

!      Print the scaled matrix

      ifail = 0
      Call x04dbf('Upper','Non-unit',n,n,a,lda,'Bracketed','F8.4',      &
        'Scaled matrix','Integer',rlabs,'Integer',clabs,80,0,ifail)

      End If
      End If

99999 Format (1X,A,I4,A)
99998 Format (1X,2(A,1P,E8.1))
99997 Format ((1X,1P,7E11.1))
      End Program f07ftfe

```

10.2 Program Data

F07FTF Example Program Data

```

4
( 3.23, 0.00) ( 1.51,-1.92) ( 1.90D+05, 0.84D+05) ( 0.42D+00, 2.50D+00) :Value of N
              ( 3.58, 0.00) (-0.23D+05, 1.11D+05) (-1.18D+00, 1.37D+00)
              ( 4.09D+10, 0.00D+00) ( 2.33D+05,-0.14D+05)
              ( 4.29D+00, 0.00D+00)
                                          :End of matrix A

```

10.3 Program Results

F07FTF Example Program Results

Matrix A

```

1      1      2      3
1 ( 3.23E+00, 0.00E+00) ( 1.51E+00, -1.92E+00) ( 1.90E+05, 8.40E+04)
2      ( 3.58E+00, 0.00E+00) ( -2.30E+04, 1.11E+05)
3      ( 4.09E+10, 0.00E+00)
4

```

```

1      4
1 ( 4.20E-01, 2.50E+00)
2 ( -1.18E+00, 1.37E+00)
3 ( 2.33E+05, -1.40E+04)
4 ( 4.29E+00, 0.00E+00)

```

SCOND = 8.9E-06, AMAX = 4.1E+10

Diagonal scaling factors

```

5.6E-01  5.3E-01  4.9E-06  4.8E-01

```

Scaled matrix

```

1      2      3
1 ( 1.0000, 0.0000) ( 0.4441, -0.5646) ( 0.5227, 0.2311)
2      ( 1.0000, 0.0000) ( -0.0601, 0.2901)
3      ( 1.0000, 0.0000)
4

```

```

1      4
1 ( 0.1128, 0.6716)
2 ( -0.3011, 0.3496)
3 ( 0.5562, -0.0334)
4 ( 1.0000, 0.0000)

```