

NAG Library Routine Document

F07CEF (DGTTRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F07CEF (DGTTRS) computes the solution to a real system of linear equations $AX = B$ or $A^T X = B$, where A is an n by n tridiagonal matrix and X and B are n by r matrices, using the LU factorization returned by F07CDF (DGTTRF).

2 Specification

```
SUBROUTINE F07CEF (TRANS, N, NRHS, DL, D, DU, DU2, IPIV, B, LDB, INFO)
INTEGER          N, NRHS, IPIV(*), LDB, INFO
REAL (KIND=nag_wp) DL(*), D(*), DU(*), DU2(*), B(LDB,*)
CHARACTER(1)     TRANS
```

The routine may be called by its LAPACK name *dgtrrs*.

3 Description

F07CEF (DGTTRS) should be preceded by a call to F07CDF (DGTTRF), which uses Gaussian elimination with partial pivoting and row interchanges to factorize the matrix A as

$$A = PLU,$$

where P is a permutation matrix, L is unit lower triangular with at most one nonzero subdiagonal element in each column, and U is an upper triangular band matrix, with two superdiagonals. F07CEF (DGTTRS) then utilizes the factorization to solve the required equations.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

5 Arguments

- 1: TRANS – CHARACTER(1) *Input*
On entry: specifies the equations to be solved as follows:
 TRANS = 'N'
 Solve $AX = B$ for X .
 TRANS = 'T' or 'C'
 Solve $A^T X = B$ for X .
Constraint: TRANS = 'N', 'T' or 'C'.
- 2: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.

- 3: NRHS – INTEGER *Input*
On entry: r , the number of right-hand sides, i.e., the number of columns of the matrix B .
Constraint: $\text{NRHS} \geq 0$.
- 4: DL(*) – REAL (KIND=nag_wp) array *Input*
Note: the dimension of the array DL must be at least $\max(1, N - 1)$.
On entry: must contain the $(n - 1)$ multipliers that define the matrix L of the LU factorization of A .
- 5: D(*) – REAL (KIND=nag_wp) array *Input*
Note: the dimension of the array D must be at least $\max(1, N)$.
On entry: must contain the n diagonal elements of the upper triangular matrix U from the LU factorization of A .
- 6: DU(*) – REAL (KIND=nag_wp) array *Input*
Note: the dimension of the array DU must be at least $\max(1, N - 1)$.
On entry: must contain the $(n - 1)$ elements of the first superdiagonal of U .
- 7: DU2(*) – REAL (KIND=nag_wp) array *Input*
Note: the dimension of the array DU2 must be at least $\max(1, N - 2)$.
On entry: must contain the $(n - 2)$ elements of the second superdiagonal of U .
- 8: IPIV(*) – INTEGER array *Input*
Note: the dimension of the array IPIV must be at least $\max(1, N)$.
On entry: must contain the n pivot indices that define the permutation matrix P . At the i th step, row i of the matrix was interchanged with row $\text{IPIV}(i)$, and $\text{IPIV}(i)$ must always be either i or $(i + 1)$, $\text{IPIV}(i) = i$ indicating that a row interchange was not performed.
- 9: B(LDB, *) – REAL (KIND=nag_wp) array *Input/Output*
Note: the second dimension of the array B must be at least $\max(1, \text{NRHS})$.
On entry: the n by r matrix of right-hand sides B .
On exit: the n by r solution matrix X .
- 10: LDB – INTEGER *Input*
On entry: the first dimension of the array B as declared in the (sub)program from which F07CEF (DGTTRS) is called.
Constraint: $\text{LDB} \geq \max(1, N)$.
- 11: INFO – INTEGER *Output*
On exit: $\text{INFO} = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

$\text{INFO} < 0$

If $\text{INFO} = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed solution for a single right-hand side, \hat{x} , satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the *machine precision*. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A) \frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

Following the use of this routine F07CGF (DGTCON) can be used to estimate the condition number of A and F07CHF (DGTRFS) can be used to obtain approximate error bounds.

8 Parallelism and Performance

F07CEF (DGTTRS) is not threaded in any implementation.

9 Further Comments

The total number of floating-point operations required to solve the equations $AX = B$ or $A^T X = B$ is proportional to nr .

The complex analogue of this routine is F07CSF (ZGTTRS).

10 Example

This example solves the equations

$$AX = B,$$

where A is the tridiagonal matrix

$$A = \begin{pmatrix} 3.0 & 2.1 & 0 & 0 & 0 \\ 3.4 & 2.3 & -1.0 & 0 & 0 \\ 0 & 3.6 & -5.0 & 1.9 & 0 \\ 0 & 0 & 7.0 & -0.9 & 8.0 \\ 0 & 0 & 0 & -6.0 & 7.1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2.7 & 6.6 \\ -0.5 & 10.8 \\ 2.6 & -3.2 \\ 0.6 & -11.2 \\ 2.7 & 19.1 \end{pmatrix}.$$

10.1 Program Text

```

Program f07cefe
!      F07CEF Example Program Text
!
!      Mark 26 Release. NAG Copyright 2016.
!
!      .. Use Statements ..
Use nag_library, Only: dgttrf, dgttrs, nag_wp, x04caf
!      .. Implicit None Statement ..
Implicit None
!      .. Parameters ..
Integer, Parameter      :: nin = 5, nout = 6
!      .. Local Scalars ..
Integer                  :: i, ifail, info, ldb, n, nrhs
!      .. Local Arrays ..
Real (Kind=nag_wp), Allocatable :: b(:,,:), d(:), dl(:), du(:), du2(:)
Integer, Allocatable     :: ipiv(:)
!      .. Executable Statements ..

```

```

Write (nout,*) 'F07CEF Example Program Results'
Write (nout,*)
Flush (nout)
! Skip heading in data file
Read (nin,*)
Read (nin,*) n, nrhs
ldb = n
Allocate (b(ldb,nrhs),d(n),dl(n-1),du(n-1),du2(n-2),ipiv(n))

! Read the tridiagonal matrix A from data file

Read (nin,*) du(1:n-1)
Read (nin,*) d(1:n)
Read (nin,*) dl(1:n-1)

! Read the right hand matrix B
Read (nin,*)(b(i,1:nrhs),i=1,n)

! Factorize the tridiagonal matrix A
! The NAG name equivalent of dgttrf is f07cdf
Call dgttrf(n,dl,d,du,du2,ipiv,info)

If (info==0) Then

! Solve the equations AX = B
! The NAG name equivalent of dgttrs is f07cef
Call dgttrs('No transpose',n,nrhs,dl,d,du,du2,ipiv,b,ldb,info)

! Print the solution

! ifail: behaviour on error exit
! =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
ifail = 0
Call x04caf('General',' ',n,nrhs,b,ldb,'Solution(s)',ifail)

Else
Write (nout,99999) 'The (', info, ', ', info, ')',
' element of the factor U is zero'
End If

99999 Format (1X,A,I3,A,I3,A,A)
End Program f07cefe

```

10.2 Program Data

```

F07CEF Example Program Data
5      2      :Values of N and NRHS
      2.1  -1.0  1.9  8.0
      3.0  2.3  -5.0  -0.9  7.1
      3.4  3.6  7.0  -6.0      :End of matrix A
      2.7  6.6
     -0.5 10.8
      2.6  -3.2
      0.6 -11.2
      2.7 19.1      :End of matrix B

```

10.3 Program Results

F07CEF Example Program Results

```

Solution(s)
      1      2
1     -4.0000  5.0000
2      7.0000 -4.0000
3      3.0000 -3.0000
4     -4.0000 -2.0000
5     -3.0000  1.0000

```