

# NAG Library Routine Document

## E04KZF

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

### 1 Purpose

E04KZF is an easy-to-use modified Newton algorithm for finding a minimum of a function  $F(x_1, x_2, \dots, x_n)$ , subject to fixed upper and lower bounds on the independent variables  $x_1, x_2, \dots, x_n$ , when first derivatives of  $F$  are available. It is intended for functions which are continuous and which have continuous first and second derivatives (although it will usually work even if the derivatives have occasional discontinuities).

### 2 Specification

```

SUBROUTINE E04KZF (N, IBOUND, FUNCT2, BL, BU, X, F, G, IW, LIW, W, LW,      &
                  IUSER, RUSER, IFAIL)
INTEGER          N, IBOUND, IW(LIW), LIW, LW, IUSER(*), IFAIL
REAL (KIND=nag_wp) BL(N), BU(N), X(N), F, G(N), W(LW), RUSER(*)
EXTERNAL        FUNCT2

```

### 3 Description

E04KZF is applicable to problems of the form:

$$\text{Minimize } F(x_1, x_2, \dots, x_n) \quad \text{subject to} \quad l_j \leq x_j \leq u_j, \quad j = 1, 2, \dots, n$$

when first derivatives are known.

Special provision is made for problems which actually have no bounds on the  $x_j$ , problems which have only non-negativity bounds, and problems in which  $l_1 = l_2 = \dots = l_n$  and  $u_1 = u_2 = \dots = u_n$ . You must supply a subroutine to calculate the values of  $F(x)$  and its first derivatives at any point  $x$ .

From a starting point you supplied there is generated, on the basis of estimates of the gradient of the curvature of  $F(x)$ , a sequence of feasible points which is intended to converge to a local minimum of the constrained function.

### 4 References

Gill P E and Murray W (1976) Minimization subject to bounds on the variables *NPL Report NAC 72* National Physical Laboratory

### 5 Arguments

1: N – INTEGER *Input*

*On entry:* the number  $n$  of independent variables.

*Constraint:*  $N \geq 1$ .

2: IBOUND – INTEGER *Input*

*On entry:* indicates whether the facility for dealing with bounds of special forms is to be used. It must be set to one of the following values:

IBOUND = 0

If you are supplying all the  $l_j$  and  $u_j$  individually.

IBOUND = 1

If there are no bounds on any  $x_j$ .

IBOUND = 2

If all the bounds are of the form  $0 \leq x_j$ .

IBOUND = 3

If  $l_1 = l_2 = \dots = l_n$  and  $u_1 = u_2 = \dots = u_n$ .

*Constraint:*  $0 \leq \text{IBOUND} \leq 3$ .

3: FUNCT2 – SUBROUTINE, supplied by the user.

*External Procedure*

You must supply this routine to calculate the values of the function  $F(x)$  and its first derivatives  $\frac{\partial F}{\partial x_j}$  at any point  $x$ . It should be tested separately before being used in conjunction with E04KZF (see Chapter E04).

The specification of FUNCT2 is:

```
SUBROUTINE FUNCT2 (N, XC, FC, GC, IUSER, RUSER)
  INTEGER          N, IUSER(*)
  REAL (KIND=nag_wp) XC(N), FC, GC(N), RUSER(*)
```

1: N – INTEGER *Input*

*On entry:* the number  $n$  of variables.

2: XC(N) – REAL (KIND=nag\_wp) array *Input*

*On entry:* the point  $x$  at which the function and derivatives are required.

3: FC – REAL (KIND=nag\_wp) *Output*

*On exit:* the value of the function  $F$  at the current point  $x$ ,

4: GC(N) – REAL (KIND=nag\_wp) array *Output*

*On exit:* GC( $j$ ) must be set to the value of the first derivative  $\frac{\partial F}{\partial x_j}$  at the point  $x$ , for  $j = 1, 2, \dots, n$ .

5: IUSER(\*) – INTEGER array *User Workspace*

6: RUSER(\*) – REAL (KIND=nag\_wp) array *User Workspace*

FUNCT2 is called with the arguments IUSER and RUSER as supplied to E04KZF. You should use the arrays IUSER and RUSER to supply information to FUNCT2.

FUNCT2 must either be a module subprogram USED by, or declared as EXTERNAL in, the (sub) program from which E04KZF is called. Arguments denoted as *Input* must **not** be changed by this procedure.

4: BL(N) – REAL (KIND=nag\_wp) array *Input/Output*

*On entry:* the lower bounds  $l_j$ .

If IBOUND is set to 0, you must set BL( $j$ ) to  $l_j$ , for  $j = 1, 2, \dots, n$ . (If a lower bound is not specified for a particular  $x_j$ , the corresponding BL( $j$ ) should be set to  $-10^6$ .)

If IBOUND is set to 3, you must set BL(1) to  $l_1$ ; E04KZF will then set the remaining elements of BL equal to BL(1).

*On exit:* the lower bounds actually used by E04KZF.

- 5: BU(N) – REAL (KIND=nag\_wp) array Input/Output  
*On entry:* the upper bounds  $u_j$ .  
 If IBOUND is set to 0, you must set BU( $j$ ) to  $u_j$ , for  $j = 1, 2, \dots, n$ . (If an upper bound is not specified for a particular  $x_j$ , the corresponding BU( $j$ ) should be set to  $10^6$ .)  
 If IBOUND is set to 3, you must set BU(1) to  $u_1$ ; E04KZF will then set the remaining elements of BU equal to BU(1).  
*On exit:* the upper bounds actually used by E04KZF.
- 6: X(N) – REAL (KIND=nag\_wp) array Input/Output  
*On entry:* X( $j$ ) must be set to a guess at the  $j$ th component of the position of the minimum, for  $j = 1, 2, \dots, n$ . The routine checks the gradient at the starting point, and is more likely to detect any error in your programming if the initial X( $j$ ) are nonzero and mutually distinct.  
*On exit:* the lowest point found during the calculations of the position of the minimum.
- 7: F – REAL (KIND=nag\_wp) Output  
*On exit:* the value of  $F(x)$  corresponding to the final point stored in X.
- 8: G(N) – REAL (KIND=nag\_wp) array Output  
*On exit:* the value of  $\frac{\partial F}{\partial x_j}$  corresponding to the final point stored in X, for  $j = 1, 2, \dots, n$ ; the value of G( $j$ ) for variables not on a bound should normally be close to zero.
- 9: IW(LIW) – INTEGER array Workspace  
 10: LIW – INTEGER Input  
*On entry:* the dimension of the array IW as declared in the (sub)program from which E04KZF is called.  
*Constraint:*  $LIW \geq N + 2$ .
- 11: W(LW) – REAL (KIND=nag\_wp) array Workspace  
 12: LW – INTEGER Input  
*On entry:* the dimension of the array W as declared in the (sub)program from which E04KZF is called.  
*Constraint:*  $LW \geq \max(N \times (N + 7), 10)$ .
- 13: IUSER(\*) – INTEGER array User Workspace  
 14: RUSER(\*) – REAL (KIND=nag\_wp) array User Workspace  
 IUSER and RUSER are not used by E04KZF, but are passed directly to FUNCT2 and should be used to pass information to this routine.
- 15: IFAIL – INTEGER Input/Output  
*On entry:* IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.  
 For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output arguments may be useful even if  $IFAIL \neq 0$  on exit, the recommended value is -1. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**  
*On exit:*  $IFAIL = 0$  unless the routine detects an error or a warning has been flagged (see Section 6).

## 6 Error Indicators and Warnings

If on entry  $IFAIL = 0$  or  $-1$ , explanatory error messages are output on the current error message unit (as defined by X04AAF).

**Note:** E04KZF may return useful information for one or more of the following detected errors or warnings.

Errors or warnings detected by the routine:

$IFAIL = 1$

On entry,  $N < 1$ ,  
 or  $IBOUND < 0$ ,  
 or  $IBOUND > 3$ ,  
 or  $IBOUND = 0$  and  $BL(j) > BU(j)$  for some  $j$ ,  
 or  $IBOUND = 3$  and  $BL(1) > BU(1)$ ,  
 or  $LIW < N + 2$ ,  
 or  $LW < \max(10, N \times (N + 7))$ .

$IFAIL = 2$

There has been a large number of function evaluations, yet the algorithm does not seem to be converging. The calculations can be restarted from the final point held in X. The error may also indicate that  $F(x)$  has no minimum.

$IFAIL = 3$

The conditions for a minimum have not all been met but a lower point could not be found and the algorithm has failed.

$IFAIL = 4$

Not used. (This value of the argument is included to make the significance of  $IFAIL = 5$  etc. consistent in the easy-to-use routines.)

$IFAIL = 5$

$IFAIL = 6$

$IFAIL = 7$

$IFAIL = 8$

There is some doubt about whether the point  $x$  found by E04KZF is a minimum. The degree of confidence in the result decreases as  $IFAIL$  increases. Thus, when  $IFAIL = 5$  it is probable that the final  $x$  gives a good estimate of the position of a minimum, but when  $IFAIL = 8$  it is very unlikely that the routine has found a minimum.

$IFAIL = 9$

In the search for a minimum, the modulus of one of the variables has become very large ( $\sim 10^6$ ). This indicates that there is a mistake in FUNCT2, that your problem has no finite solution, or that the problem needs rescaling (see Section 9).

$IFAIL = 10$

It is very likely that you have made an error in forming the gradient.

$IFAIL = -99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

IFAIL = -399

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

IFAIL = -999

Dynamic memory allocation failed.

See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

If you are dissatisfied with the result (e.g., because IFAIL = 5, 6, 7 or 8), it is worth restarting the calculations from a different starting point (not the point at which the failure occurred) in order to avoid the region which caused the failure. If persistent trouble occurs and it is possible to calculate second derivatives it may be advisable to change to a routine which uses second derivatives (see the E04 Chapter Introduction).

## 7 Accuracy

When a successful exit is made then, for a computer with a mantissa of  $t$  decimals, one would expect to get about  $t/2 - 1$  decimals accuracy in  $x$  and about  $t - 1$  decimals accuracy in  $F$ , provided the problem is reasonably well scaled.

## 8 Parallelism and Performance

E04KZF is not threaded in any implementation.

## 9 Further Comments

The number of iterations required depends on the number of variables, the behaviour of  $F(x)$  and the distance of the starting point from the solution. The number of operations performed in an iteration of E04KZF is roughly proportional to  $n^3 + O(n^2)$ . In addition, each iteration makes at least  $m + 1$  calls of FUNCT2 where  $m$  is the number of variables not fixed on bounds. So unless  $F(x)$  and the gradient vector can be evaluated very quickly, the run time will be dominated by the time spent in FUNCT2.

Ideally the problem should be scaled so that at the solution the value of  $F(x)$  and the corresponding values of  $x_1, x_2, \dots, x_n$  are in the range  $(-1, +1)$ , and so that at points a unit distance away from the solution,  $F$  is approximately a unit value greater than at the minimum. It is unlikely that you will be able to follow these recommendations very closely, but it is worth trying (by guesswork), as sensible scaling will reduce the difficulty of the minimization problem, so that E04KZF will take less computer time.

## 10 Example

A program to minimize

$$F = (x_1 + 10x_2)^2 + 5(x_3 - x_4)^2 + (x_2 - 2x_3)^4 + 10(x_1 - x_4)^4$$

subject to

$$\begin{array}{rcccl} 1 & \leq & x_1 & \leq & 3 \\ -2 & \leq & x_2 & \leq & 0 \\ 1 & \leq & x_4 & \leq & 3 \end{array}$$

starting from the initial guess  $(3, -1, 0, 1)$ .

In practice, it is worth trying to make FUNCT2 as efficient as possible. This has not been done in the example program for reasons of clarity.

## 10.1 Program Text

```

!   E04KZF Example Program Text
!   Mark 26 Release. NAG Copyright 2016.
!   Module e04kzfe_mod

!       E04KZF Example Program Module:
!           Parameters and User-defined Routines

!       .. Use Statements ..
!       Use nag_library, Only: nag_wp
!       .. Implicit None Statement ..
!       Implicit None
!       .. Accessibility Statements ..
!       Private
!       Public                                :: funct2
!       .. Parameters ..
!       Integer, Parameter, Public           :: n = 4, nout = 6
!       Integer, Parameter, Public           :: liw = n + 2
!       Integer, Parameter, Public           :: lw = n*(n+7)
!   Contains
!       Subroutine funct2(n,xc,fc,gc,iuser,ruser)
!           Routine to evaluate objective function and its 1st derivatives.

!           .. Scalar Arguments ..
!           Real (Kind=nag_wp), Intent (Out) :: fc
!           Integer, Intent (In)             :: n
!           .. Array Arguments ..
!           Real (Kind=nag_wp), Intent (Out) :: gc(n)
!           Real (Kind=nag_wp), Intent (Inout) :: ruser(*)
!           Real (Kind=nag_wp), Intent (In) :: xc(n)
!           Integer, Intent (Inout)         :: iuser(*)
!           .. Local Scalars ..
!           Real (Kind=nag_wp)              :: x1, x2, x3, x4
!           .. Executable Statements ..
!           x1 = xc(1)
!           x2 = xc(2)
!           x3 = xc(3)
!           x4 = xc(4)
!           fc = (x1+10.0_nag_wp*x2)**2 + 5.0_nag_wp*(x3-x4)**2 +
!               (x2-2.0_nag_wp*x3)**4 + 10.0_nag_wp*(x1-x4)**4
!           gc(1) = 2.0_nag_wp*(x1+10.0_nag_wp*x2) + 40.0_nag_wp*(x1-x4)**3
!           gc(2) = 20.0_nag_wp*(x1+10.0_nag_wp*x2) + 4.0_nag_wp*(x2-2.0_nag_wp*x3 &
!               )**3
!           gc(3) = 10.0_nag_wp*(x3-x4) - 8.0_nag_wp*(x2-2.0_nag_wp*x3)**3
!           gc(4) = -10.0_nag_wp*(x3-x4) - 40.0_nag_wp*(x1-x4)**3

!           Return

!       End Subroutine funct2
!   End Module e04kzfe_mod
!   Program e04kzfe

!       E04KZF Example Main Program

!       .. Use Statements ..
!       Use nag_library, Only: e04kzf, nag_wp
!       Use e04kzfe_mod, Only: funct2, liw, lw, n, nout
!       .. Implicit None Statement ..
!       Implicit None
!       .. Local Scalars ..
!       Real (Kind=nag_wp)                :: f
!       Integer                            :: ibound, ifail
!       .. Local Arrays ..
!       Real (Kind=nag_wp)                 :: bl(n), bu(n), g(n), ruser(1), w(liw), &
!           x(n)
!       Integer                            :: iuser(1), iw(liw)
!       .. Executable Statements ..
!       Write (nout,*) 'E04KZF Example Program Results'
!       Flush (nout)

```

```

x(1:n) = (/3.0_nag_wp,-1.0_nag_wp,0.0_nag_wp,1.0_nag_wp/)
ibound = 0
! X(3) is unconstrained, so we set BL(3) to a large negative
! number and BU(3) to a large positive number.
bl(1:n) = (/1.0_nag_wp,-2.0_nag_wp,-1.0E6_nag_wp,1.0_nag_wp/)
bu(1:n) = (/3.0_nag_wp,0.0_nag_wp,1.0E6_nag_wp,3.0_nag_wp/)
ifail = -1
Call e04kzf(n,ibound,funct2,bl,bu,x,f,g,iw,liw,w,lw,iuser,ruser,ifail)
Select Case (ifail)
Case (0,2:)
  Write (nout,*)
  Write (nout,99999) 'Function value on exit is ', f
  Write (nout,99999) 'at the point', x(1:n)
  Write (nout,*) 'the corresponding (machine dependent) gradient is'
  Write (nout,99998) g(1:n)
End Select
99999 Format (1X,A,4F12.4)
99998 Format (13X,4E12.4)
End Program e04kzfe

```

## 10.2 Program Data

None.

## 10.3 Program Results

```

E04KZF Example Program Results
** It is probable that a local minimum has been found,
** but it cannot be guaranteed.
** ABNORMAL EXIT from NAG Library routine E04KZF: IFAIL =      5
** NAG soft failure - control returned

Function value on exit is      2.4338
at the point      1.0000      -0.0852      0.4093      1.0000
the corresponding (machine dependent) gradient is
      0.2953E+00 -0.5870E-09  0.1176E-08  0.5907E+01

```

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