

NAG Library Function Document

nag_fresnel_s (s20acc)

1 Purpose

nag_fresnel_s (s20acc) returns a value for the Fresnel integral $S(x)$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_fresnel_s (double x)
```

3 Description

nag_fresnel_s (s20acc) evaluates an approximation to the Fresnel integral

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right) dt.$$

Note: $S(x) = -S(-x)$, so the approximation need only consider $x \geq 0.0$.

The function is based on three Chebyshev expansions:

For $0 < x \leq 3$,

$$S(x) = x^3 \sum_{r=0} a_r T_r(t), \quad \text{with } t = 2\left(\frac{x}{3}\right)^4 - 1.$$

For $x > 3$,

$$S(x) = \frac{1}{2} - \frac{f(x)}{x} \cos\left(\frac{\pi}{2}x^2\right) - \frac{g(x)}{x^3} \sin\left(\frac{\pi}{2}x^2\right),$$

where $f(x) = \sum_{r=0} b_r T_r(t)$,

and $g(x) = \sum_{r=0} c_r T_r(t)$,

with $t = 2\left(\frac{3}{x}\right)^4 - 1$.

For small x , $S(x) \simeq \frac{\pi}{6}x^3$. This approximation is used when x is sufficiently small for the result to be correct to **machine precision**. For very small x , this approximation would underflow; the result is then set exactly to zero.

For large x , $f(x) \simeq \frac{1}{\pi}$ and $g(x) \simeq \frac{1}{\pi^2}$. Therefore for moderately large x , when $\frac{1}{\pi^2 x^3}$ is negligible compared with $\frac{1}{2}$, the second term in the approximation for $x > 3$ may be dropped. For very large x , when $\frac{1}{\pi x}$ becomes negligible, $S(x) \simeq \frac{1}{2}$. However there will be considerable difficulties in calculating $\cos\left(\frac{\pi}{2}x^2\right)$ accurately before this final limiting value can be used. Since $\cos\left(\frac{\pi}{2}x^2\right)$ is periodic, its value is essentially determined by the fractional part of x^2 . If $x^2 = N + \theta$ where N is an integer and $0 \leq \theta < 1$, then $\cos\left(\frac{\pi}{2}x^2\right)$ depends on θ and on N modulo 4. By exploiting this fact, it is possible to

retain significance in the calculation of $\cos\left(\frac{\pi}{2}x^2\right)$ either all the way to the very large x limit, or at least until the integer part of $\frac{x}{2}$ is equal to the maximum integer allowed on the machine.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: **x** – double

Input

On entry: the argument x of the function.

6 Error Indicators and Warnings

None.

7 Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively.

If δ is somewhat larger than the *machine precision* (i.e., if δ is due to data errors etc.), then ϵ and δ are approximately related by:

$$\epsilon \simeq \left| \frac{x \sin\left(\frac{\pi}{2}x^2\right)}{S(x)} \right| \delta.$$

Figure 1 shows the behaviour of the error amplification factor $\left| \frac{x \sin\left(\frac{\pi}{2}x^2\right)}{S(x)} \right|$.

However if δ is of the same order as the *machine precision*, then rounding errors could make ϵ slightly larger than the above relation predicts.

For small x , $\epsilon \simeq 3\delta$ and hence there is only moderate amplification of relative error. Of course for very small x where the correct result would underflow and exact zero is returned, relative error-control is lost.

For moderately large values of x ,

$$|\epsilon| \simeq \left| 2x \sin\left(\frac{\pi}{2}x^2\right) \right| |\delta|$$

and the result will be subject to increasingly large amplification of errors. However the above relation breaks down for large values of x (i.e., when $\frac{1}{x^2}$ is of the order of the *machine precision*); in this region the relative error in the result is essentially bounded by $\frac{2}{\pi x}$.

Hence the effects of error amplification are limited and at worst the relative error loss should not exceed half the possible number of significant figures.

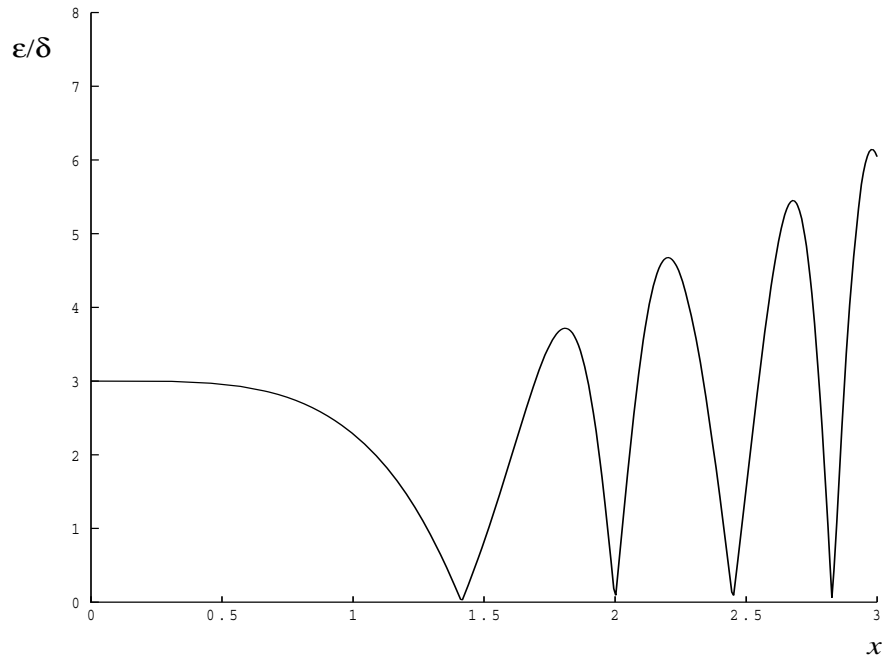


Figure 1

8 Parallelism and Performance

nag_fresnel_s (s20acc) is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```

/* nag_fresnel_s (s20acc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer exit_status = 0;
    double x, y;

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else

```

```

    scanf("%*[\n]");
#endif
    printf("nag_fresnel_s (s20acc) Example Program Results\n");
    printf("      x          y\n");
#ifdef _WIN32
    while (scanf_s("%lf", &x) != EOF)
#else
    while (scanf("%lf", &x) != EOF)
#endif
    {
        /* nag_fresnel_s (s20acc).
         * Fresnel integral S(x)
         */
        y = nag_fresnel_s(x);
        printf("%12.3e%12.3e\n", x, y);
    }

    return exit_status;
}

```

10.2 Program Data

```

nag_fresnel_s (s20acc) Example Program Data
      0.0
      0.5
      1.0
      2.0
      4.0
      5.0
      6.0
      8.0
     10.0
     -1.0
    1000.0

```

10.3 Program Results

```

nag_fresnel_s (s20acc) Example Program Results
      x          y
  0.000e+00  0.000e+00
  5.000e-01  6.473e-02
  1.000e+00  4.383e-01
  2.000e+00  3.434e-01
  4.000e+00  4.205e-01
  5.000e+00  4.992e-01
  6.000e+00  4.470e-01
  8.000e+00  4.602e-01
  1.000e+01  4.682e-01
 -1.000e+00 -4.383e-01
  1.000e+03  4.997e-01

```

Example Program
Returns a Value for the Fresnel Integral $S(x)$

