NAG Library Function Document

nag_tsa_multi_diff (g13dlc)

1 Purpose

nag_tsa_multi_diff (g13dlc) differences and/or transforms a multivariate time series.

2 Specification

```
#include <nag.h>
#include <nagg13.h>
void nag_tsa_multi_diff (Integer k, Integer n, const double z[],
     const Integer tr[], const Integer id[], const double delta[],
     double w[], Integer *nd, NagError *fail)
```

3 Description

For certain time series it may first be necessary to difference the original data to obtain a stationary series before calculating autocorrelations, etc. This function also allows you to apply either a square root or a log transformation to the original time series to stabilize the variance if required.

If the order of differencing required for the ith series is d_i, then the differencing operator is defined by $\delta_i(B) = 1 - \delta_{i1}B - \delta_{i2}B^2 - \cdots - \delta_{id_i}B^{d_i}$, where B is the backward shift operator; that is, $BZ_t = Z_{t-1}$. Let d denote the maximum of the orders of differencing, d_i, over the k series. The function computes values of the differenced/transformed series $W_t = (w_{1t}, w_{2t}, \ldots, w_{kt})^{\mathrm{T}}$, for $t = d+1, \ldots, n$, as follows:

$$w_{it} = \delta_i(B)z^*_{it}, \quad i = 1, 2, \ldots, k$$

where z^*_{it} are the transformed values of the original k-dimensional time series $Z_t = (z_{1t}, z_{2t}, \ldots, z_{kt})^{\mathrm{T}}$.

The differencing parameters δ_{ij}, for $i = 1, 2, \ldots, k$ and $j = 1, 2, \ldots, d_i$, must be supplied by you. If the ith series does not require differencing, then $d_i = 0$.

4 References

Box G E P and Jenkins G M (1976) *Time Series Analysis: Forecasting and Control* (Revised Edition) Holden–Day

Wei W W S (1990) *Time Series Analysis: Univariate and Multivariate Methods* Addison–Wesley

5 Arguments

1: **k** – Integer *Input*

On entry: k, the dimension of the multivariate time series.

Constraint: $\mathbf{k} \geq 1$.

2: **n** – Integer *Input*

On entry: n, the number of observations in the series, prior to differencing.

Constraint: $\mathbf{n} \geq 1$.

3: **z**[**k** × **n**] – const double *Input*

On entry: $\mathbf{z}[(t-1)k + i - 1]$ must contain the ith series at time t, for $t = 1, 2, \ldots, n$ and $i = 1, 2, \ldots, k$.

4: **tr**[**k**] – const Integer *Input*

On entry: **tr**$[i-1]$ indicates whether the ith series is to be transformed, for $i = 1, 2, \ldots, k$.

tr$[i-1] = -1$
A square root transformation is used.

tr$[i-1] = 0$
No transformation is used.

tr$[i-1] = 1$
A log transformation is used.

Constraint: **tr**$[i-1] = -1$, 0 or 1, for $i = 1, 2, \ldots, k$.

5: **id**[**k**] – const Integer *Input*

On entry: the order of differencing for each series, $d_1, d_2, \ldots, d_k$.

Constraint: $0 \leq$ **id**$[i] <$ **n**, for $i = 0, 1, \ldots,$ **k** $- 1$.

6: **delta**$[dim]$ – const double *Input*

Note: the dimension, *dim*, of the array **delta** must be at least **k** $\times \max(1, d)$, where $d = \max($**id**$[i-1])$.

On entry: if **id**$[i-1] > 0$ then **delta**$[(j-1)k + i - 1]$ must be set to δ_{ij}, for $j = 1, 2, \ldots, d_l$ and $i = 1, 2, \ldots, k$.

7: **w**$[dim]$ – double *Output*

Note: the dimension, *dim*, of the array **w** must be at least **k** $\times$ (**n** $- d$), where $d = \max($**id**$[i-1])$.

On exit: **w**$[(t-1)k + i - 1]$ contains the value of $w_{i,t+d}$, for $i = 1, 2, \ldots, k$ and $t = 1, 2, \ldots, n - d$.

8: **nd** – Integer * *Output*

On exit: the number of differenced values, $n - d$, in the series, where $d = \max($**id**$[i-1])$.

9: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **k** $=$ ⟨*value*⟩.
Constraint: **k** ≥ 1.

On entry, **n** $=$ ⟨*value*⟩.
Constraint: **n** ≥ 1.

NE_INT_ARRAY

On entry, element $\langle value \rangle$ of **id** is greater than or equal to **n**.

On entry, element $\langle value \rangle$ of **id** is less than zero.

On entry, $\mathbf{tr}[\langle value \rangle] = \langle value \rangle$.
Constraint: $\mathbf{tr}[i] = -1,\ 0$ or 1.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

NE_TRANSFORMATION

On entry, one (or more) of the transformations requested is invalid.

7 Accuracy

The computations are believed to be stable.

8 Parallelism and Performance

nag_tsa_multi_diff (g13dlc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The same differencing operator does not have to be applied to all the series. For example, suppose we have $k = 2$, and wish to apply the second-order differencing operator ∇^2 to the first series and the first-order differencing operator ∇ to the second series:

$$w_{1t} = \nabla^2 z_{1t} = (1-B)^2 z_{1t} = \left(1 - 2B + B^2\right) z_{1t}, \qquad \text{and}$$

$$w_{2t} = \nabla z_{2t} = (1-B) z_{2t}.$$

Then $d_1 = 2, d_2 = 1,\ d = \max(d_1, d_2) = 2,$ and

$$\mathbf{delta} = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & \end{bmatrix}.$$

10 Example

A program to difference (non-seasonally) each of two time series of length 48. No transformation is to be applied to either of the series.

10.1 Program Text

```
/* nag_tsa_multi_diff (g13dlc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg13.h>

int main(void)
{
  /* Scalars */
  Integer exit_status, i, j, k, maxd, mind, n, nd, nw, pdw, pddelta, kmax;
  NagError fail;

  /* Arrays */
  double *delta = 0, *w = 0, *z = 0;
  Integer *id = 0, *tr = 0;

#define W(I, J)     w[(J-1)*pdw + I - 1]
#define DELTA(I, J) delta[(J-1)*pddelta + I - 1]
#define Z(I, J)     z[(J-1)*kmax + I - 1]

  INIT_FAIL(fail);

  exit_status = 0;

  printf("nag_tsa_multi_diff (g13dlc) Example Program Results\n");

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif

#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%" NAG_IFMT "%*[^\n] ", &k, &n);
#else
  scanf("%" NAG_IFMT "%" NAG_IFMT "%*[^\n] ", &k, &n);
#endif

  if (k > 0 && n > 0) {
    kmax = k;
    /* Allocate array id */
    if (!(id = NAG_ALLOC(k, Integer)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

    for (i = 1; i <= k; ++i)
#ifdef _WIN32
      scanf_s("%" NAG_IFMT "", &id[i - 1]);
#else
      scanf("%" NAG_IFMT "", &id[i - 1]);
#endif
#ifdef _WIN32
    scanf_s("%*[^\n] ");
#else
    scanf("%*[^\n] ");
#endif
```

```
    mind = 0;
    maxd = 0;
    for (i = 1; i <= k; ++i) {
      mind = MIN(mind, id[i - 1]);
      maxd = MAX(maxd, id[i - 1]);
    }

    if (mind >= 0) {
      /* Allocate arrays */
      nw = n - maxd;
      if (!(tr = NAG_ALLOC(k, Integer)) ||
          !(delta = NAG_ALLOC(kmax * maxd, double)) ||
          !(w = NAG_ALLOC(kmax * nw, double)) ||
          !(z = NAG_ALLOC(kmax * n, double)))
      {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
      }

      pdw = kmax;
      pddelta = kmax;

      for (i = 1; i <= k; ++i) {
        for (j = 1; j <= n; ++j)
#ifdef _WIN32
          scanf_s("%lf", &Z(i, j));
#else
          scanf("%lf", &Z(i, j));
#endif
#ifdef _WIN32
        scanf_s("%*[^\n] ");
#else
        scanf("%*[^\n] ");
#endif
      }

      for (i = 1; i <= k; ++i)
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "", &tr[i - 1]);
#else
        scanf("%" NAG_IFMT "", &tr[i - 1]);
#endif
#ifdef _WIN32
      scanf_s("%*[^\n] ");
#else
      scanf("%*[^\n] ");
#endif

      if (maxd > 0) {
        for (i = 1; i <= k; ++i) {
          for (j = 1; j <= id[i - 1]; ++j)
#ifdef _WIN32
            scanf_s("%lf", &DELTA(i, j));
#else
            scanf("%lf", &DELTA(i, j));
#endif
#ifdef _WIN32
          scanf_s("%*[^\n] ");
#else
          scanf("%*[^\n] ");
#endif
        }
      }

      /* nag_tsa_multi_diff (g13dlc).
       * Multivariate time series, differences and/or transforms
       */
      nag_tsa_multi_diff(k, n, z, tr, id, delta, w, &nd, &fail);

      if (fail.code != NE_NOERROR) {
```

```
        printf("Error from nag_tsa_multi_diff (g13dlc).\n%s\n", fail.message);
        exit_status = 1;
        goto END;
      }
      printf("\n");
      printf(" Transformed/Differenced series\n");
      printf(" ------------------------------\n");

      for (i = 1; i <= k; ++i) {
        printf("\n");
        printf(" Series %2" NAG_IFMT "\n", i);
        printf(" -----------\n");
        printf("\n");
        printf(" Number of differenced values = %6" NAG_IFMT "\n", nd);
        printf("\n");
        for (j = 1; j <= nd; ++j) {
          printf("%10.3f", W(i, j));
          if (j % 8 == 0 || j == nd)
            printf("\n");
        }
      }
    }
  }

END:
  NAG_FREE(tr);
  NAG_FREE(delta);
  NAG_FREE(w);
  NAG_FREE(z);
  NAG_FREE(id);

  return exit_status;
}
```

10.2 Program Data

```
nag_tsa_multi_diff (g13dlc) Example Program Data
 2 48   :  k, n
 1  1   :  id(0), id(1)
-1.490 -1.620  5.200  6.230  6.210  5.860  4.090  3.180
 2.620  1.490  1.170  0.850 -0.350  0.240  2.440  2.580
 2.040  0.400  2.260  3.340  5.090  5.000  4.780  4.110
 3.450  1.650  1.290  4.090  6.320  7.500  3.890  1.580
 5.210  5.250  4.930  7.380  5.870  5.810  9.680  9.070
 7.290  7.840  7.550  7.320  7.970  7.760  7.000  8.350
 7.340  6.350  6.960  8.540  6.620  4.970  4.550  4.810
 4.750  4.760 10.880 10.010 11.620 10.360  6.400  6.240
 7.930  4.040  3.730  5.600  5.350  6.810  8.270  7.680
 6.650  6.080 10.250  9.140 17.750 13.300  9.630  6.800
 4.080  5.060  4.940  6.650  7.940 10.760 11.890  5.850
 9.010  7.500 10.020 10.380  8.150  8.370 10.730 12.140 : End of time series
 0  0  :  tr(0), tr(1)
 1.0   :  delta(1,1)
 1.0   :  delta(2,1)
```

10.3 Program Results

```
nag_tsa_multi_diff (g13dlc) Example Program Results

 Transformed/Differenced series
 ------------------------------

 Series  1
 -----------

 Number of differenced values =     47

    -0.130     6.820     1.030    -0.020    -0.350    -1.770    -0.910    -0.560
    -1.130    -0.320    -0.320    -1.200     0.590     2.200     0.140    -0.540
    -1.640     1.860     1.080     1.750    -0.090    -0.220    -0.670    -0.660
```

```
   -1.800    -0.360     2.800     2.230     1.180    -3.610    -2.310     3.630
    0.040    -0.320     2.450    -1.510    -0.060     3.870    -0.610    -1.780
    0.550    -0.290    -0.230     0.650    -0.210    -0.760     1.350

 Series  2
 -----------

 Number of differenced values =     47

   -0.990     0.610     1.580    -1.920    -1.650    -0.420     0.260    -0.060
    0.010     6.120    -0.870     1.610    -1.260    -3.960    -0.160     1.690
   -3.890    -0.310     1.870    -0.250     1.460     1.460    -0.590    -1.030
   -0.570     4.170    -1.110     8.610    -4.450    -3.670    -2.830    -2.720
    0.980    -0.120     1.710     1.290     2.820     1.130    -6.040     3.160
   -1.510     2.520     0.360    -2.230     0.220     2.360     1.410
```