NAG Library Function Document

nag_bivariate_students_t (g01hcc)

1 Purpose

nag_bivariate_students_t (g01hcc) returns probabilities for the bivariate Student's t-distribution.

2 Specification

```
#include <nag.h>
#include <nagg01.h>
double nag_bivariate_students_t (Nag_TailProbability tail, const double a[],
       const double b[], Integer df, double rho, NagError *fail)
```

3 Description

Let the vector random variable $X = (X_1, X_2)^{\mathrm{T}}$ follow a bivariate Student's t-distribution with degrees of freedom ν and correlation ρ, then the probability density function is given by

$$f\left(X : \nu, \rho\right) = \frac{1}{2\pi\sqrt{1-\rho^2}}\left(1 + \frac{X_1^2 + X_2^2 - 2\rho X_1 X_2}{\nu(1-\rho^2)}\right)^{-\nu/2-1}.$$

The lower tail probability is defined by:

$$P\left(X_1 \leq b_1, X_2 \leq b_2 : \nu, \rho\right) = \int_{-\infty}^{b_1}\int_{-\infty}^{b_2} f\left(X : \nu, \rho\right) dX_2 dX_1.$$

The upper tail probability is defined by:

$$P\left(X_1 \geq a_1, X_2 \geq a_2 : \nu, \rho\right) = \int_{a_1}^{\infty}\int_{a_2}^{\infty} f\left(X : \nu, \rho\right) dX_2 dX_1.$$

The central probability is defined by:

$$P\left(a_1 \leq X_1 \leq b_1, a_2 \leq X_2 \leq b_2 : \nu, \rho\right) = \int_{a_1}^{b_1}\int_{a_2}^{b_2} f\left(X : \nu, \rho\right) dX_2 dX_1.$$

Calculations use the Dunnet and Sobel (1954) method, as described by Genz (2004).

4 References

Dunnet C W and Sobel M (1954) A bivariate generalization of Student's t-distribution, with tables for certain special cases *Biometrika* **41** 153–169

Genz A (2004) Numerical computation of rectangular bivariate and trivariate Normal and t probabilities *Statistics and Computing* **14** 151–160

5 Arguments

1: **tail** – Nag_TailProbability *Input*

On entry: indicates which probability is to be returned.

tail = Nag_LowerTail
The lower tail probability is returned.

tail = Nag_UpperTail
The upper tail probability is returned.

tail = Nag_Central
The central probability is returned.

Constraint: **tail** = Nag_LowerTail, Nag_UpperTail or Nag_Central.

2: **a**[**2**] – const double *Input*

On entry: if **tail** = Nag_Central or Nag_UpperTail, the lower bounds a_1 and a_2.

If **tail** = Nag_LowerTail, **a** is not referenced.

3: **b**[**2**] – const double *Input*

On entry: if **tail** = Nag_Central or Nag_LowerTail, the upper bounds b_1 and b_2.

If **tail** = Nag_UpperTail, **b** is not referenced.

Constraint: if **tail** = Nag_Central, $a_i < b_i$, for $i = 1, 2$.

4: **df** – Integer *Input*

On entry: ν, the degrees of freedom of the bivariate Student's t-distribution.

Constraint: **df** ≥ 1.

5: **rho** – double *Input*

On entry: ρ, the correlation of the bivariate Student's t-distribution.

Constraint: $-1.0 \leq$ **rho** ≤ 1.0.

6: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **df** = ⟨*value*⟩.
Constraint: **df** ≥ 1.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

NE_REAL

On entry, **rho** $= \langle value \rangle$.
Constraint: $-1.0 \leq$ **rho** ≤ 1.0.

NE_REAL_2

On entry, $\mathbf{b}[i-1] \leq \mathbf{a}[i-1]$ for central probability, for some $i = 1, 2$.

7 Accuracy

Accuracy of the algorithm implemented here is discussed in comparison with algorithms based on a generalized Placket formula by Genz (2004), who recommends the Dunnet and Sobel method. This implementation should give a maximum absolute error of the order of 10^{-16}.

8 Parallelism and Performance

nag_bivariate_students_t (g01hcc) is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example calculates the bivariate Student's t probability given the choice of tail and degrees of freedom, correlation and bounds.

10.1 Program Text

```
/* nag_bivariate_students_t (g01hcc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */
#include <stdio.h>
#include <string.h>
#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
  /* Scalars */
  Integer df, exit_status = 0, ierr;
  double prob, rho;
  /* Arrays */
  char nag_enum_arg[14];
  double a[2], b[2];
  /* NAG types */
  Nag_TailProbability tail;
  NagError fail;

  printf("%s\n\n",
         "nag_bivariate_students_t (g01hcc) Example Program Results");

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
```

```
  scanf("%*[^\n]");
#endif

  /* Display headers */
  printf("%-8s%2s%-8s%2s%-8s%2s%-8s%2s%-4s%2s%-8s%2s%-14s%2s%-8s\n\n",
         "a1", " ", "b1", " ", "a2", " ", "b2", " ", "df", " ", "rho", " ",
         "Tail", " ", "p");

  while (1) {
#ifdef _WIN32
    ierr = scanf_s("%13s", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    ierr = scanf("%13s", nag_enum_arg);
#endif
    if (ierr == EOF || ierr < 1) {
      break;
    }

    /* Initialize limits */
    a[0] = a[1] = b[0] = b[1] = 0.0;

    /* nag_enum_name_to_value (x04nac).
     * Converts NAG enum member name to value
     */
    tail = (Nag_TailProbability) nag_enum_name_to_value(nag_enum_arg);

    /* Read parameter values */
    switch (tail) {
    case Nag_LowerTail:
#ifdef _WIN32
      scanf_s("%" NAG_IFMT "%lf%lf%lf", &df, &rho, b, b + 1);
#else
      scanf("%" NAG_IFMT "%lf%lf%lf", &df, &rho, b, b + 1);
#endif
      break;
    case Nag_Central:
#ifdef _WIN32
      scanf_s("%" NAG_IFMT "%lf%lf%lf%lf%lf", &df, &rho, a, b, a + 1, b + 1);
#else
      scanf("%" NAG_IFMT "%lf%lf%lf%lf%lf", &df, &rho, a, b, a + 1, b + 1);
#endif
      break;
    case Nag_UpperTail:
#ifdef _WIN32
      scanf_s("%" NAG_IFMT "%lf%lf%lf", &df, &rho, a, a + 1);
#else
      scanf("%" NAG_IFMT "%lf%lf%lf", &df, &rho, a, a + 1);
#endif
      break;
    default:
      printf(" %s\n", "Invalid tail specification in data file");
      exit_status = -1;
      goto END;
    }

#ifdef _WIN32
    scanf_s("%*[^\n]");
#else
    scanf("%*[^\n]");
#endif

    /* Calculate probability for the bivariate Student's t-distribution */
    INIT_FAIL(fail);
    /* nag_bivariate_students_t (g01hcc) */
    prob = nag_bivariate_students_t(tail, a, b, df, rho, &fail);

    /* Display results */
    switch (tail) {
    case Nag_LowerTail:
      printf("%-8s%2s%-8g%2s%-8s%2s%-8g",
             "-Inf", " ", b[0], " ", "-Inf", " ", b[1]);
```

```
      break;
    case Nag_Central:
      printf("%-8g%2s%-8g%2s%-8g%2s%-8g",
             a[0], " ", b[0], " ", a[1], " ", b[1]);
      break;
    case Nag_UpperTail:
      printf("%-8g%2s%-8s%2s%-8g%2s%-8s",
             a[0], " ", "Inf", " ", a[1], " ", "Inf");
      break;
    default:
      {
        printf("Invalid tail specification.\n");
        exit_status = -1;
        goto END;
      }
    }

    printf("%2s%-4" NAG_IFMT "%2s%-8g%2s%-14s%2s%-8.4f\n",
           " ", df, " ", rho, " ", nag_enum_arg, " ", prob);
  }

END:
  return exit_status;
}
```

10.2 Program Data

```
nag_bivariate_students_t (g01hcc) Example Program Data
Nag_LowerTail  8  0.6   4.0 0.8         : tail df rho      b[i], i=0,1
Nag_Central   12 -0.2 -40.0 2.0 0.0 4.0 : tail df rho (a, b)[i], i=0,1
Nag_UpperTail  2  0.3  -2.0 8.0         : tail df rho      a[i], i=0,1
```

10.3 Program Results

```
nag_bivariate_students_t (g01hcc) Example Program Results

a1        b1        a2        b2        df    rho       Tail            p

-Inf      4         -Inf      0.8       8     0.6       Nag_LowerTail   0.7764
-40       2         0         4         12    -0.2      Nag_Central     0.4876
-2        Inf       8         Inf       2     0.3       Nag_UpperTail   0.0059
```