NAG Library Function Document

nag_dgbbrd (f08lec)

1 Purpose

nag_dgbbrd (f08lec) reduces a real m by n band matrix to upper bidiagonal form.

2 Specification

```
#include <nag.h>
#include <nagf08.h>
```

```
void nag_dgbbrd (Nag_OrderType order, Nag_VectType vect, Integer m,
     Integer n, Integer ncc, Integer kl, Integer ku, double ab[],
     Integer pdab, double d[], double e[], double q[], Integer pdq,
     double pt[], Integer pdpt, double c[], Integer pdc, NagError *fail)
```

3 Description

nag_dgbbrd (f08lec) reduces a real m by n band matrix to upper bidiagonal form B by an orthogonal transformation: $A = QBP^{\mathrm{T}}$. The orthogonal matrices Q and P^{T}, of order m and n respectively, are determined as a product of Givens rotation matrices, and may be formed explicitly by the function if required. A matrix C may also be updated to give $\tilde{C} = Q^{\mathrm{T}}C$.

The function uses a vectorizable form of the reduction.

4 References

None.

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **vect** – Nag_VectType *Input*

On entry: indicates whether the matrices Q and/or P^{T} are generated.

vect = Nag_DoNotForm
Neither Q nor P^{T} is generated.

vect = Nag_FormQ
Q is generated.

vect = Nag_FormP
P^{T} is generated.

vect = Nag_FormBoth
Both Q and P^{T} are generated.

Constraint: **vect** = Nag_DoNotForm, Nag_FormQ, Nag_FormP or Nag_FormBoth.

3: **m** – Integer *Input*

On entry: m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

4: **n** – Integer *Input*

On entry: n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5: **ncc** – Integer *Input*

On entry: n_C, the number of columns of the matrix C.

Constraint: $\mathbf{ncc} \geq 0$.

6: **kl** – Integer *Input*

On entry: the number of subdiagonals, k_l, within the band of A.

Constraint: $\mathbf{kl} \geq 0$.

7: **ku** – Integer *Input*

On entry: the number of superdiagonals, k_u, within the band of A.

Constraint: $\mathbf{ku} \geq 0$.

8: **ab**[*dim*] – double *Input/Output*

Note: the dimension, *dim*, of the array **ab** must be at least

$\max(1, \mathbf{pdab} \times \mathbf{n})$ when **order** = Nag_ColMajor;
$\max(1, \mathbf{m} \times \mathbf{pdab})$ when **order** = Nag_RowMajor.

On entry: the original m by n band matrix A.

This is stored as a notional two-dimensional array with row elements or column elements stored contiguously. The storage of elements A_{ij}, for row $i = 1, \ldots, m$ and column $j = \max(1, i - k_l), \ldots, \min(n, i + k_u)$, depends on the **order** argument as follows:

if **order** = Nag_ColMajor, A_{ij} is stored as $\mathbf{ab}[(j-1) \times \mathbf{pdab} + \mathbf{ku} + i - j]$;

if **order** = Nag_RowMajor, A_{ij} is stored as $\mathbf{ab}[(i-1) \times \mathbf{pdab} + \mathbf{kl} + j - i]$.

On exit: **ab** is overwritten by values generated during the reduction.

9: **pdab** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix A in the array **ab**.

Constraint: $\mathbf{pdab} \geq \mathbf{kl} + \mathbf{ku} + 1$.

10: $\mathbf{d}[\mathbf{min}(\mathbf{m}, \mathbf{n})]$ – double *Output*

On exit: the diagonal elements of the bidiagonal matrix B.

11: $\mathbf{e}[\mathbf{min}(\mathbf{m}, \mathbf{n}) - \mathbf{1}]$ – double *Output*

On exit: the superdiagonal elements of the bidiagonal matrix B.

12: **q**[*dim*] – double *Output*

Note: the dimension, *dim*, of the array **q** must be at least

max(1, **pdq** × **m**) when **vect** = Nag_FormQ or Nag_FormBoth;
1 otherwise.

The (i,j)th element of the matrix Q is stored in

q[$(j-1)$ × **pdq** + $i-1$] when **order** = Nag_ColMajor;
q[$(i-1)$ × **pdq** + $j-1$] when **order** = Nag_RowMajor.

On exit: if **vect** = Nag_FormQ or Nag_FormBoth, contains the m by m orthogonal matrix Q.

If **vect** = Nag_DoNotForm or Nag_FormP, **q** is not referenced.

13: **pdq** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **q**.

Constraints:

if **vect** = Nag_FormQ or Nag_FormBoth, **pdq** $\geq$ max(1, **m**);
otherwise **pdq** ≥ 1.

14: **pt**[*dim*] – double *Output*

Note: the dimension, *dim*, of the array **pt** must be at least

max(1, **pdpt** × **n**) when **vect** = Nag_FormP or Nag_FormBoth;
1 otherwise.

The (i,j)th element of the matrix is stored in

pt[$(j-1)$ × **pdpt** + $i-1$] when **order** = Nag_ColMajor;
pt[$(i-1)$ × **pdpt** + $j-1$] when **order** = Nag_RowMajor.

On exit: the n by n orthogonal matrix P^{T}, if **vect** = Nag_FormP or Nag_FormBoth. If **vect** = Nag_DoNotForm or Nag_FormQ, **pt** is not referenced.

15: **pdpt** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **pt**.

Constraints:

if **vect** = Nag_FormP or Nag_FormBoth, **pdpt** $\geq$ max(1, **n**);
otherwise **pdpt** ≥ 1.

16: **c**[*dim*] – double *Input/Output*

Note: the dimension, *dim*, of the array **c** must be at least

max(1, **pdc** × **ncc**) when **order** = Nag_ColMajor;
max(1, **m** × **pdc**) when **order** = Nag_RowMajor.

The (i,j)th element of the matrix C is stored in

c[$(j-1)$ × **pdc** + $i-1$] when **order** = Nag_ColMajor;
c[$(i-1)$ × **pdc** + $j-1$] when **order** = Nag_RowMajor.

On entry: an m by n_C matrix C.

On exit: **c** is overwritten by $Q^{\mathrm{T}}C$. If **ncc** = 0, **c** is not referenced.

17: **pdc** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **c**.

Constraints:

if **order** $=$ Nag_ColMajor,

if **ncc** > 0, **pdc** $\geq \max(1, \mathbf{m})$;
if **ncc** $= 0$, **pdc** ≥ 1.;

if **order** $=$ Nag_RowMajor, **pdc** $\geq \max(1, \mathbf{ncc})$.

18: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_ENUM_INT_2

On entry, **vect** $= \langle value \rangle$, **pdpt** $= \langle value \rangle$ and **n** $= \langle value \rangle$.
Constraint: if **vect** $=$ Nag_FormP or Nag_FormBoth, **pdpt** $\geq \max(1, \mathbf{n})$;
otherwise **pdpt** ≥ 1.

On entry, **vect** $= \langle value \rangle$, **pdq** $= \langle value \rangle$ and **m** $= \langle value \rangle$.
Constraint: if **vect** $=$ Nag_FormQ or Nag_FormBoth, **pdq** $\geq \max(1, \mathbf{m})$;
otherwise **pdq** ≥ 1.

NE_INT

On entry, **kl** $= \langle value \rangle$.
Constraint: **kl** ≥ 0.

On entry, **ku** $= \langle value \rangle$.
Constraint: **ku** ≥ 0.

On entry, **m** $= \langle value \rangle$.
Constraint: **m** ≥ 0.

On entry, **n** $= \langle value \rangle$.
Constraint: **n** ≥ 0.

On entry, **ncc** $= \langle value \rangle$.
Constraint: **ncc** ≥ 0.

On entry, **pdab** $= \langle value \rangle$.
Constraint: **pdab** > 0.

On entry, **pdc** $= \langle value \rangle$.
Constraint: **pdc** > 0.

On entry, **pdpt** $= \langle value \rangle$.
Constraint: **pdpt** > 0.

On entry, **pdq** $= \langle value \rangle$.
Constraint: **pdq** > 0.

NE_INT_2

On entry, **pdc** $= \langle value \rangle$ and **ncc** $= \langle value \rangle$.
Constraint: **pdc** $\geq \max(1, \mathbf{ncc})$.

NE_INT_3

On entry, **ncc** $= \langle value \rangle$, **pdc** $= \langle value \rangle$ and **m** $= \langle value \rangle$.
Constraint: if **ncc** > 0, **pdc** $\geq \max(1, \mathbf{m})$;
if **ncc** $= 0$, **pdc** ≥ 1.

On entry, **pdab** $= \langle value \rangle$, **kl** $= \langle value \rangle$ and **ku** $= \langle value \rangle$.
Constraint: **pdab** $\geq \mathbf{kl} + \mathbf{ku} + 1$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

The computed bidiagonal form B satisfies $QBP^{\mathrm{T}} = A + E$, where

$$\|E\|_2 \leq c(n)\epsilon\|A\|_2,$$

$c(n)$ is a modestly increasing function of n, and ϵ is the ***machine precision***.

The elements of B themselves may be sensitive to small perturbations in A or to rounding errors in the computation, but this does not affect the stability of the singular values and vectors.

The computed matrix Q differs from an exactly orthogonal matrix by a matrix F such that

$$\|F\|_2 = O(\epsilon).$$

A similar statement holds for the computed matrix P^{T}.

8 Parallelism and Performance

nag_dgbbrd (f08lec) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The total number of real floating-point operations is approximately the sum of:

$6n^2k$, if **vect** = Nag_DoNotForm and **ncc** $= 0$, and

$3n^2n_C(k-1)/k$, if C is updated, and

$3n^3(k-1)/k$, if either Q or P^T is generated (double this if both),

where $k = k_l + k_u$, assuming $n \gg k$. For this section we assume that $m = n$.

The complex analogue of this function is nag_zgbbrd (f08lsc).

10 Example

This example reduces the matrix A to upper bidiagonal form, where

$$A = \begin{pmatrix} -0.57 & -1.28 & 0.00 & 0.00 \\ -1.93 & 1.08 & -0.31 & 0.00 \\ 2.30 & 0.24 & 0.40 & -0.35 \\ 0.00 & 0.64 & -0.66 & 0.08 \\ 0.00 & 0.00 & 0.15 & -2.13 \\ -0.00 & 0.00 & 0.00 & 0.50 \end{pmatrix}.$$

10.1 Program Text

```
/* nag_dgbbrd (f08lec) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf08.h>
#include <nagf16.h>
#include <nagx02.h>
#include <nagx04.h>

int main(void)
{
  /* Scalars */
  double alpha, beta, norm;
  Integer i, j, kl, ku, m, n, ncc, pdab, pdaw, pdc, pdf, pdq;
  Integer pdpt, d_len, e_len;
  Integer exit_status = 0;
  NagError fail;
  Nag_OrderType order;
  /* Arrays */
  double *ab = 0, *aw = 0, *c = 0, *d = 0, *e = 0, *f = 0, *pt = 0, *q = 0;

#ifdef NAG_COLUMN_MAJOR
#define AB(I, J) ab[(J - 1) * pdab + ku + I - J]
#define AW(I, J) aw[(J - 1) * pdaw + I - 1]
#define F(I, J) f[(J - 1) * pdf + I - 1]
#define Q(I, J) q[(J - 1) * pdq + I - 1]
  order = Nag_ColMajor;
#else
#define AB(I, J) ab[(I - 1) * pdab + kl + J - I]
#define AW(I, J) aw[(I - 1) * pdaw + J - 1]
#define F(I, J) f[(I - 1) * pdf + J - 1]
#define Q(I, J) q[(I - 1) * pdq + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);

  printf("nag_dgbbrd (f08lec) Example Program Results\n");

  /* Skip heading in data file */
```

```
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif
#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT
          "%*[^\n] ", &m, &n, &kl, &ku, &ncc);
#else
  scanf("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT
        "%*[^\n] ", &m, &n, &kl, &ku, &ncc);
#endif
#ifdef NAG_COLUMN_MAJOR
  pdab = kl + ku + 1;
  pdaw = m;
  pdf = m;
  pdq = m;
  pdpt = n;
  pdc = m;
#else
  pdab = kl + ku + 1;
  pdaw = n;
  pdf = n;
  pdq = m;
  pdpt = n;
  pdc = MAX(1, ncc);
#endif
  d_len = MIN(m, n);
  e_len = MIN(m, n) - 1;

  /* Allocate memory */
  if (!(ab = NAG_ALLOC((kl + ku + 1) * m, double)) ||
      !(aw = NAG_ALLOC(m * n, double)) ||
      !(f = NAG_ALLOC(m * n, double)) ||
      !(c = NAG_ALLOC(m * MAX(1, ncc), double)) ||
      !(d = NAG_ALLOC(d_len, double)) ||
      !(e = NAG_ALLOC(e_len, double)) ||
      !(pt = NAG_ALLOC(n * n, double)) || !(q = NAG_ALLOC(m * m, double)))
  {
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
  }

  /* Read A from data file */
  for (i = 1; i <= m; ++i) {
    for (j = MAX(1, i - kl); j <= MIN(n, i + ku); ++j)
#ifdef _WIN32
      scanf_s("%lf", &AB(i, j));
#else
      scanf("%lf", &AB(i, j));
#endif
  }
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif

  /* Copy AB into AW */
  for (i = 1; i <= m; ++i) {
    for (j = 1; j <= n; ++j) {
      if (j >= MAX(1, i - kl) && j <= MIN(n, i + ku))
        AW(i, j) = AB(i, j);
      else
        AW(i, j) = 0;
    }
  }

  /* nag_gen_real_mat_print (x04cac): Print Matrix A. */
  fflush(stdout);
  nag_gen_real_mat_print(order, Nag_GeneralMatrix, Nag_NonUnitDiag, m, n,
                         aw, pdaw, "Matrix A", 0, &fail);
  printf("\n");
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_real_mat_print (x04cac).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }

  /* Reduce A to bidiagonal form */
  /* nag_dgbbrd (f08lec).
   * Reduction of real rectangular band matrix to upper
   * bidiagonal form
   */
  nag_dgbbrd(order, Nag_FormBoth, m, n, ncc, kl, ku, ab,
             pdab, d, e, q, pdq, pt, pdpt, c, pdc, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_dgbbrd (f08lec).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }

  /* F = Q*B */
  for (i = 1; i <= m; i++) {
    F(i, 1) = Q(i, 1) * d[0];
    for (j = 2; j <= n; j++) {
      F(i, j) = (Q(i, j) * d[j - 1]) + (Q(i, j - 1) * e[j - 2]);
    }
  }

  /* nag_dgemm (f16yac): Compute A - Q*B*P^T from the factorization of A */
  /* and store in matrix AW */
  alpha = -1.0;
  beta = 1.0;
  nag_dgemm(order, Nag_NoTrans, Nag_NoTrans, m, n, n, alpha, f, pdf,
            pt, pdpt, beta, aw, pdaw, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_dgemm (f16yac).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }

  /* nag_dge_norm (f16rac): Find norm of matrix AW and print warning if */
  /* it is too large */
  nag_dge_norm(order, Nag_OneNorm, m, n, aw, pdaw, &norm, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_dge_norm (f16rac).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }
  if (norm > pow(x02ajc(), 0.8)) {
    printf("%s\n%s\n", "Norm of A-(Q*B*P^T) is much greater than 0.",
           "Schur factorization has failed.");
  }

END:
  NAG_FREE(ab);
  NAG_FREE(aw);
  NAG_FREE(c);
  NAG_FREE(d);
  NAG_FREE(e);
  NAG_FREE(f);
  NAG_FREE(pt);
  NAG_FREE(q);

  return exit_status;
}
```

10.2 Program Data

```
nag_dgbbrd (f08lec) Example Program Data
  6  4  2  1  0                 :Values of M, N, KL, KU and NCC
 -0.57  -1.28
 -1.93   1.08  -0.31
  2.30   0.24   0.40  -0.35
         0.64  -0.66   0.08
                0.15  -2.13
                       0.50   :End of matrix A
```

10.3 Program Results

```
nag_dgbbrd (f08lec) Example Program Results
 Matrix A
            1          2          3          4
 1     -0.5700    -1.2800     0.0000     0.0000
 2     -1.9300     1.0800    -0.3100     0.0000
 3      2.3000     0.2400     0.4000    -0.3500
 4      0.0000     0.6400    -0.6600     0.0800
 5      0.0000     0.0000     0.1500    -2.1300
 6      0.0000     0.0000     0.0000     0.5000
```