

# NAG Library Function Document

## nag\_zhpsv (f07pnc)

### 1 Purpose

nag\_zhpsv (f07pnc) computes the solution to a complex system of linear equations

$$AX = B,$$

where  $A$  is an  $n$  by  $n$  Hermitian matrix stored in packed format and  $X$  and  $B$  are  $n$  by  $r$  matrices.

### 2 Specification

```
#include <nag.h>
#include <nagf07.h>

void nag_zhpsv (Nag_OrderType order, Nag_UploType uplo, Integer n,
               Integer nrhs, Complex ap[], Integer ipiv[], Complex b[], Integer pdb,
               NagError *fail)
```

### 3 Description

nag\_zhpsv (f07pnc) uses the diagonal pivoting method to factor  $A$  as  $A = UDU^H$  if **uplo** = Nag\_Upper or  $A = LDL^H$  if **uplo** = Nag\_Lower, where  $U$  (or  $L$ ) is a product of permutation and unit upper (lower) triangular matrices,  $D$  is Hermitian and block diagonal with 1 by 1 and 2 by 2 diagonal blocks. The factored form of  $A$  is then used to solve the system of equations  $AX = B$ .

### 4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J (2002) *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

### 5 Arguments

1: **order** – Nag\_OrderType *Input*

*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.

2: **uplo** – Nag\_UploType *Input*

*On entry:* if **uplo** = Nag\_Upper, the upper triangle of  $A$  is stored.

If **uplo** = Nag\_Lower, the lower triangle of  $A$  is stored.

*Constraint:* **uplo** = Nag\_Upper or Nag\_Lower.

3: **n** – Integer Input

*On entry:*  $n$ , the number of linear equations, i.e., the order of the matrix  $A$ .

*Constraint:*  $n \geq 0$ .

4: **nrhs** – Integer Input

*On entry:*  $r$ , the number of right-hand sides, i.e., the number of columns of the matrix  $B$ .

*Constraint:*  $nrhs \geq 0$ .

5: **ap**[*dim*] – Complex Input/Output

**Note:** the dimension, *dim*, of the array **ap** must be at least  $\max(1, n \times (n + 1)/2)$ .

*On entry:* the  $n$  by  $n$  Hermitian matrix  $A$ , packed by rows or columns.

The storage of elements  $A_{ij}$  depends on the **order** and **uplo** arguments as follows:

- if **order** = Nag\_ColMajor and **uplo** = Nag\_Upper,  
 $A_{ij}$  is stored in **ap**[( $j - 1$ )  $\times$   $j/2 + i - 1$ ], for  $i \leq j$ ;
- if **order** = Nag\_ColMajor and **uplo** = Nag\_Lower,  
 $A_{ij}$  is stored in **ap**[( $2n - j$ )  $\times$  ( $j - 1$ )/2 +  $i - 1$ ], for  $i \geq j$ ;
- if **order** = Nag\_RowMajor and **uplo** = Nag\_Upper,  
 $A_{ij}$  is stored in **ap**[( $2n - i$ )  $\times$  ( $i - 1$ )/2 +  $j - 1$ ], for  $i \leq j$ ;
- if **order** = Nag\_RowMajor and **uplo** = Nag\_Lower,  
 $A_{ij}$  is stored in **ap**[( $i - 1$ )  $\times$   $i/2 + j - 1$ ], for  $i \geq j$ .

*On exit:* the block diagonal matrix  $D$  and the multipliers used to obtain the factor  $U$  or  $L$  from the factorization  $A = UDU^H$  or  $A = LDL^H$  as computed by nag\_zhptrf (f07prc), stored as a packed triangular matrix in the same storage format as  $A$ .

6: **ipiv**[**n**] – Integer Output

*On exit:* details of the interchanges and the block structure of  $D$ . More precisely,

if **ipiv**[ $i - 1$ ] =  $k > 0$ ,  $d_{ii}$  is a 1 by 1 pivot block and the  $i$ th row and column of  $A$  were interchanged with the  $k$ th row and column;

if **uplo** = Nag\_Upper and **ipiv**[ $i - 2$ ] = **ipiv**[ $i - 1$ ] =  $-l < 0$ ,  $\begin{pmatrix} d_{i-1,i-1} & \bar{d}_{i,i-1} \\ \bar{d}_{i,i-1} & d_{ii} \end{pmatrix}$  is a 2 by 2 pivot block and the ( $i - 1$ )th row and column of  $A$  were interchanged with the  $l$ th row and column;

if **uplo** = Nag\_Lower and **ipiv**[ $i - 1$ ] = **ipiv**[ $i$ ] =  $-m < 0$ ,  $\begin{pmatrix} d_{ii} & d_{i+1,i} \\ d_{i+1,i} & d_{i+1,i+1} \end{pmatrix}$  is a 2 by 2 pivot block and the ( $i + 1$ )th row and column of  $A$  were interchanged with the  $m$ th row and column.

7: **b**[*dim*] – Complex Input/Output

**Note:** the dimension, *dim*, of the array **b** must be at least

$\max(1, \mathbf{pdb} \times \mathbf{nrhs})$  when **order** = Nag\_ColMajor;

$\max(1, n \times \mathbf{pdb})$  when **order** = Nag\_RowMajor.

The ( $i, j$ )th element of the matrix  $B$  is stored in

**b**[( $j - 1$ )  $\times$  **pdb** +  $i - 1$ ] when **order** = Nag\_ColMajor;

**b**[( $i - 1$ )  $\times$  **pdb** +  $j - 1$ ] when **order** = Nag\_RowMajor.

*On entry:* the  $n$  by  $r$  right-hand side matrix  $B$ .

*On exit:* if **fail.code** = NE\_NOERROR, the  $n$  by  $r$  solution matrix  $X$ .

- 8: **pdb** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **b**.  
*Constraints:*  
 if **order** = Nag\_ColMajor, **pdb**  $\geq$  max(1, **n**);  
 if **order** = Nag\_RowMajor, **pdb**  $\geq$  max(1, **nrhs**).

- 9: **fail** – NagError \* *Input/Output*  
 The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.  
 See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INT

On entry, **n** =  $\langle value \rangle$ .

Constraint: **n**  $\geq$  0.

On entry, **nrhs** =  $\langle value \rangle$ .

Constraint: **nrhs**  $\geq$  0.

On entry, **pdb** =  $\langle value \rangle$ .

Constraint: **pdb**  $>$  0.

### NE\_INT\_2

On entry, **pdb** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .

Constraint: **pdb**  $\geq$  max(1, **n**).

On entry, **pdb** =  $\langle value \rangle$  and **nrhs** =  $\langle value \rangle$ .

Constraint: **pdb**  $\geq$  max(1, **nrhs**).

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

### NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

### NE\_SINGULAR

Element  $\langle value \rangle$  of the diagonal is exactly zero. The factorization has been completed, but the block diagonal matrix *D* is exactly singular, so the solution could not be computed.

## 7 Accuracy

The computed solution for a single right-hand side,  $\hat{x}$ , satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and  $\epsilon$  is the *machine precision*. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A) \frac{\|E\|_1}{\|A\|_1},$$

where  $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$ , the condition number of  $A$  with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) and Chapter 11 of Higham (2002) for further details.

nag\_zhpsvx (f07ppc) is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, nag\_herm\_packed\_lin\_solve (f04cjc) solves  $AX = B$  and returns a forward error bound and condition estimate. nag\_herm\_packed\_lin\_solve (f04cjc) calls nag\_zhpsv (f07pnc) to solve the equations.

## 8 Parallelism and Performance

nag\_zhpsv (f07pnc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The total number of floating-point operations is approximately  $\frac{4}{3}n^3 + 8n^2r$ , where  $r$  is the number of right-hand sides.

The real analogue of this function is nag\_dspsv (f07pac). The complex symmetric analogue of this function is nag\_zspsv (f07qnc).

## 10 Example

This example solves the equations

$$Ax = b,$$

where  $A$  is the Hermitian matrix

$$A = \begin{pmatrix} -1.84 & 0.11 - 0.11i & -1.78 - 1.18i & 3.91 - 1.50i \\ 0.11 + 0.11i & -4.63 & -1.84 + 0.03i & 2.21 + 0.21i \\ -1.78 + 1.18i & -1.84 - 0.03i & -8.87 & 1.58 - 0.90i \\ 3.91 + 1.50i & 2.21 - 0.21i & 1.58 + 0.90i & -1.36 \end{pmatrix}$$

and

$$b = \begin{pmatrix} 2.98 - 10.18i \\ -9.58 + 3.88i \\ -0.77 - 16.05i \\ 7.79 + 5.48i \end{pmatrix}.$$

Details of the factorization of  $A$  are also output.

## 10.1 Program Text

```

/* nag_zhpsv (f07pnc) Example Program.
*
* NAGPRODCODE Version.
*
* Copyright 2016 Numerical Algorithms Group.
*
* Mark 26, 2016.
*/

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>

int main(void)
{
    /* Scalars */
    Integer exit_status = 0, i, j, n, nrhs, pdb;

    /* Arrays */
    Complex *ap = 0, *b = 0;
    Integer *ipiv = 0;
    char nag_enum_arg[40];

    /* Nag Types */
    NagError fail;
    Nag_UploType uplo;
    Nag_OrderType order;

#ifdef NAG_COLUMN_MAJOR
#define A_UPPER(I, J) ap[J*(J-1)/2 + I - 1]
#define A_LOWER(I, J) ap[(2*n-J)*(J-1)/2 + I - 1]
#define B(I, J)      b[(J-1)*pdb + I - 1]
    order = Nag_ColMajor;
#else
#define A_LOWER(I, J) ap[I*(I-1)/2 + J - 1]
#define A_UPPER(I, J) ap[(2*n-I)*(I-1)/2 + J - 1]
#define B(I, J)      b[(I-1)*pdb + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_zhpsv (f07pnc) Example Program Results\n\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif

#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%*[\n]", &n, &nrhs);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%*[\n]", &n, &nrhs);
#endif
    if (n < 0 || nrhs < 0) {
        printf("Invalid n or nrhs\n");
        exit_status = 1;
        goto END;
    }
#ifdef _WIN32
    scanf_s(" %39s%*[\n]", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf(" %39s%*[\n]", nag_enum_arg);
#endif
    /* nag_enum_name_to_value (x04nac).

```

```

    * Converts NAG enum member name to value
    */
    uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);

    /* Allocate memory */
    if (!(ap = NAG_ALLOC(n * (n + 1) / 2, Complex)) ||
        !(b = NAG_ALLOC(n * nrhs, Complex)) || !(ipiv = NAG_ALLOC(n, Integer)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
#ifdef NAG_COLUMN_MAJOR
    pdb = n;
#else
    pdb = nrhs;
#endif

    /* Read the triangular part of the matrix A from data file */
    if (uplo == Nag_Upper)
        for (i = 1; i <= n; ++i)
            for (j = i; j <= n; ++j)
#ifdef _WIN32
                scanf_s(" ( %lf , %lf )", &A_UPPER(i, j).re, &A_UPPER(i, j).im);
#else
                scanf(" ( %lf , %lf )", &A_UPPER(i, j).re, &A_UPPER(i, j).im);
#endif
    else if (uplo == Nag_Lower)
        for (i = 1; i <= n; ++i)
            for (j = 1; j <= i; ++j)
#ifdef _WIN32
                scanf_s(" ( %lf , %lf )", &A_LOWER(i, j).re, &A_LOWER(i, j).im);
#else
                scanf(" ( %lf , %lf )", &A_LOWER(i, j).re, &A_LOWER(i, j).im);
#endif
    #ifdef _WIN32
        scanf_s("%*[\n]");
    #else
        scanf("%*[\n]");
    #endif

    /* Read b from data file */
    for (i = 1; i <= n; ++i)
        for (j = 1; j <= nrhs; ++j)
#ifdef _WIN32
            scanf_s(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
#else
            scanf(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
#endif
    #ifdef _WIN32
        scanf_s("%*[\n]");
    #else
        scanf("%*[\n]");
    #endif

    /* Solve the equations Ax = b for x using nag_zhpsv (f07pnc). */
    nag_zhpsv(order, uplo, n, nrhs, ap, ipiv, b, pdb, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_zhpsv (f07pnc).\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }

    /* Print solution */
    printf("      Solution\n");
    for (i = 1; i <= n; ++i) {
        for (j = 1; j <= nrhs; ++j)
            printf(" (%7.4f, %7.4f)%s", B(i, j).re, B(i, j).im,
                j % 4 == 0 ? "\n" : "");
        printf("\n");
    }

```

```

END:
  NAG_FREE(ap);
  NAG_FREE(b);
  NAG_FREE(ipiv);

  return exit_status;
}

#undef A_UPPER
#undef A_LOWER
#undef B

```

## 10.2 Program Data

```

nag_zhpsv (f07pnc) Example Program Data
  4          1          : n, nrhs
  Nag_Lower          : uplo
  ( -1.84,  0.00)
  (  0.11,  0.11) ( -4.63,  0.00)
  ( -1.78,  1.18) ( -1.84, -0.03) ( -8.87,  0.00)
  (  3.91,  1.50) (  2.21, -0.21) (  1.58,  0.90) ( -1.36 ,  0.00) : matrix A
  (  2.98,-10.18) ( -9.58,  3.88) ( -0.77,-16.05) (  7.79,  5.48) : vector b

```

## 10.3 Program Results

```

nag_zhpsv (f07pnc) Example Program Results

  Solution
  ( 2.0000,  1.0000)
  ( 3.0000, -2.0000)
  (-1.0000,  2.0000)
  ( 1.0000, -1.0000)

```

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