

# NAG Library Function Document

## nag\_zgttrf (f07crc)

### 1 Purpose

nag\_zgttrf (f07crc) computes the  $LU$  factorization of a complex  $n$  by  $n$  tridiagonal matrix  $A$ .

### 2 Specification

```
#include <nag.h>
#include <nagf07.h>

void nag_zgttrf (Integer n, Complex dl[], Complex d[], Complex du[],
                Complex du2[], Integer ipiv[], NagError *fail)
```

### 3 Description

nag\_zgttrf (f07crc) uses Gaussian elimination with partial pivoting and row interchanges to factorize the matrix  $A$  as

$$A = PLU,$$

where  $P$  is a permutation matrix,  $L$  is unit lower triangular with at most one nonzero subdiagonal element in each column, and  $U$  is an upper triangular band matrix, with two superdiagonals.

### 4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

### 5 Arguments

- 1: **n** – Integer *Input*  
*On entry:*  $n$ , the order of the matrix  $A$ .  
*Constraint:*  $n \geq 0$ .
- 2: **dl**[*dim*] – Complex *Input/Output*  
**Note:** the dimension, *dim*, of the array **dl** must be at least  $\max(1, n - 1)$ .  
*On entry:* must contain the  $(n - 1)$  subdiagonal elements of the matrix  $A$ .  
*On exit:* is overwritten by the  $(n - 1)$  multipliers that define the matrix  $L$  of the  $LU$  factorization of  $A$ .
- 3: **d**[*dim*] – Complex *Input/Output*  
**Note:** the dimension, *dim*, of the array **d** must be at least  $\max(1, n)$ .  
*On entry:* must contain the  $n$  diagonal elements of the matrix  $A$ .  
*On exit:* is overwritten by the  $n$  diagonal elements of the upper triangular matrix  $U$  from the  $LU$  factorization of  $A$ .
- 4: **du**[*dim*] – Complex *Input/Output*  
**Note:** the dimension, *dim*, of the array **du** must be at least  $\max(1, n - 1)$ .

*On entry:* must contain the  $(n - 1)$  superdiagonal elements of the matrix  $A$ .

*On exit:* is overwritten by the  $(n - 1)$  elements of the first superdiagonal of  $U$ .

5: **du2[n - 2]** – Complex *Output*

*On exit:* contains the  $(n - 2)$  elements of the second superdiagonal of  $U$ .

6: **ipiv[n]** – Integer *Output*

*On exit:* contains the  $n$  pivot indices that define the permutation matrix  $P$ . At the  $i$ th step, row  $i$  of the matrix was interchanged with row **ipiv**[ $i - 1$ ]. **ipiv**[ $i - 1$ ] will always be either  $i$  or  $(i + 1)$ , **ipiv**[ $i - 1$ ] =  $i$  indicating that a row interchange was not performed.

7: **fail** – NagError \* *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INT

On entry,  $\mathbf{n} = \langle value \rangle$ .

Constraint:  $\mathbf{n} \geq 0$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

### NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

### NE\_SINGULAR

Element  $\langle value \rangle$  of the diagonal is exactly zero. The factorization has been completed, but the factor  $U$  is exactly singular, and division by zero will occur if it is used to solve a system of equations.

## 7 Accuracy

The computed factorization satisfies an equation of the form

$$A + E = PLU,$$

where

$$\|E\|_{\infty} = O(\epsilon)\|A\|_{\infty}$$

and  $\epsilon$  is the *machine precision*.

Following the use of this function, `nag_zgttrs` (f07csc) can be used to solve systems of equations  $AX = B$  or  $A^T X = B$  or  $A^H X = B$ , and `nag_zgtcon` (f07cuc) can be used to estimate the condition number of  $A$ .

## 8 Parallelism and Performance

`nag_zgttrf` (f07crc) is not threaded in any implementation.

## 9 Further Comments

The total number of floating-point operations required to factorize the matrix  $A$  is proportional to  $n$ .

The real analogue of this function is `nag_dgttrf` (f07cdc).

## 10 Example

This example factorizes the tridiagonal matrix  $A$  given by

$$A = \begin{pmatrix} -1.3 + 1.3i & 2.0 - 1.0i & 0 & 0 & 0 & 0 \\ 1.0 - 2.0i & -1.3 + 1.3i & 2.0 + 1.0i & 0 & 0 & 0 \\ 0 & 1.0 + 1.0i & -1.3 + 3.3i & -1.0 + 1.0i & 0 & 0 \\ 0 & 0 & 2.0 - 3.0i & -0.3 + 4.3i & 1.0 - 1.0i & 0 \\ 0 & 0 & 0 & 1.0 + 1.0i & -3.3 + 1.3i & 0 \end{pmatrix}.$$

### 10.1 Program Text

```

/* nag_zgttrf (f07crc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>

int main(void)
{
    /* Scalars */
    Integer exit_status = 0, i, n;

    /* Arrays */
    Complex *d = 0, *dl = 0, *du = 0, *du2 = 0;
    Integer *ipiv = 0;

    /* Nag Types */
    NagError fail;

    INIT_FAIL(fail);

    printf("nag_zgttrf (f07crc) Example Program Results\n\n");
    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else

```

```

    scanf("%*[\n]");
#endif

#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[\n]", &n);
#else
    scanf("%" NAG_IFMT "%*[\n]", &n);
#endif
if (n < 0) {
    printf("Invalid n\n");
    exit_status = 1;
    goto END;
}

/* Allocate memory */
if (!(d = NAG_ALLOC(n, Complex)) ||
    !(dl = NAG_ALLOC(n - 1, Complex)) ||
    !(du = NAG_ALLOC(n - 1, Complex)) ||
    !(du2 = NAG_ALLOC(n - 2, Complex)) || !(ipiv = NAG_ALLOC(n, Integer)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read the tridiagonal matrix A from data file */
#ifdef _WIN32
    for (i = 0; i < n - 1; ++i)
        scanf_s(" ( %lf , %lf )", &du[i].re, &du[i].im);
#else
    for (i = 0; i < n - 1; ++i)
        scanf(" ( %lf , %lf )", &du[i].re, &du[i].im);
#endif
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
#ifdef _WIN32
    for (i = 0; i < n; ++i)
        scanf_s(" ( %lf , %lf )", &d[i].re, &d[i].im);
#else
    for (i = 0; i < n; ++i)
        scanf(" ( %lf , %lf )", &d[i].re, &d[i].im);
#endif
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
#ifdef _WIN32
    for (i = 0; i < n - 1; ++i)
        scanf_s(" ( %lf , %lf )", &dl[i].re, &dl[i].im);
#else
    for (i = 0; i < n - 1; ++i)
        scanf(" ( %lf , %lf )", &dl[i].re, &dl[i].im);
#endif
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif

/* Factorize the tridiagonal matrix A using nag_zgttrf (f07crc). */
nag_zgttrf(n, dl, d, du, du2, ipiv, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_zgttrf (f07crc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
}

```

```

/* Print details of the factorization */
printf("Details of factorization (U)\n\n");

printf("%17s%24s%22s\n", "Main diagonal", "First superdiagonal",
      "Second superdiagonal");
for (i = 0; i < n; ++i) {
    printf("(%8.4f, %8.4f)", d[i].re, d[i].im);
    if (i < n - 1)
        printf(" (%8.4f, %8.4f)", du[i].re, du[i].im);
    if (i < n - 2)
        printf(" (%8.4f, %8.4f)", du2[i].re, du2[i].im);
    printf("\n");
}

printf("\n Multipliers\n");
for (i = 0; i < n - 1; ++i)
    printf("(%8.4f, %8.4f)\n", dl[i].re, dl[i].im);

printf("\n Vector of interchanges\n");
for (i = 0; i < n; ++i)
    printf("%7" NAG_IFMT "%s", ipiv[i], i % 5 == 4 ? "\n" : " ");
printf("\n");
END:
NAG_FREE(d);
NAG_FREE(dl);
NAG_FREE(du);
NAG_FREE(du2);
NAG_FREE(ipiv);

return exit_status;
}

```

## 10.2 Program Data

nag\_zgttrf (f07crc) Example Program Data

```

5
      ( 2.0,-1.0) ( 2.0, 1.0) (-1.0, 1.0) ( 1.0,-1.0) : n
(-1.3, 1.3) (-1.3, 1.3) (-1.3, 3.3) (-0.3, 4.3) (-3.3, 1.3) : du
( 1.0,-2.0) ( 1.0, 1.0) ( 2.0,-3.0) ( 1.0, 1.0) : dl

```

## 10.3 Program Results

nag\_zgttrf (f07crc) Example Program Results

Details of factorization (U)

```

      Main diagonal      First superdiagonal      Second superdiagonal
( 1.0000, -2.0000) ( -1.3000,  1.3000) ( 2.0000,  1.0000)
( 1.0000,  1.0000) ( -1.3000,  3.3000) ( -1.0000,  1.0000)
( 2.0000, -3.0000) ( -0.3000,  4.3000) ( 1.0000, -1.0000)
( 1.0000,  1.0000) ( -3.3000,  1.3000)
( -1.3399,  0.2875)

```

Multipliers

```

( -0.7800, -0.2600)
(  0.1620, -0.4860)
( -0.0452, -0.0010)
( -0.3979, -0.0562)

```

Vector of interchanges

```

      2      3      4      5      5

```