

# NAG Library Routine Document

## S30NBF

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

### 1 Purpose

S30NBF computes the European option price given by Heston's stochastic volatility model together with its sensitivities (Greeks).

### 2 Specification

```

SUBROUTINE S30NBF (CALPUT, M, N, X, S, T, SIGMAV, KAPPA, CORR, VARO,      &
                  ETA, GRISK, R, Q, P, LDP, DELTA, GAMMA, VEGA, THETA,    &
                  RHO, VANNA, CHARM, SPEED, ZOMMA, VOMMA, IFAIL)
INTEGER          M, N, LDP, IFAIL
REAL (KIND=nag_wp) X(M), S, T(N), SIGMAV, KAPPA, CORR, VARO, ETA,      &
                  GRISK, R, Q, P(LDP,N), DELTA(LDP,N), GAMMA(LDP,N),    &
                  VEGA(LDP,N), THETA(LDP,N), RHO(LDP,N),                &
                  VANNA(LDP,N), CHARM(LDP,N), SPEED(LDP,N),            &
                  ZOMMA(LDP,N), VOMMA(LDP,N)
CHARACTER(1)    CALPUT

```

### 3 Description

S30NBF computes the price and sensitivities of a European option using Heston's stochastic volatility model. The return on the asset price,  $S$ , is

$$\frac{dS}{S} = (r - q)dt + \sqrt{v_t}dW_t^{(1)}$$

and the instantaneous variance,  $v_t$ , is defined by a mean-reverting square root stochastic process,

$$dv_t = \kappa(\eta - v_t)dt + \sigma_v\sqrt{v_t}dW_t^{(2)},$$

where  $r$  is the risk free annual interest rate;  $q$  is the annual dividend rate;  $v_t$  is the variance of the asset price;  $\sigma_v$  is the volatility of the volatility,  $\sqrt{v_t}$ ;  $\kappa$  is the mean reversion rate;  $\eta$  is the long term variance.  $dW_t^{(i)}$ , for  $i = 1, 2$ , denotes two correlated standard Brownian motions with

$$\text{Cov}\left[dW_t^{(1)}, dW_t^{(2)}\right] = \rho dt.$$

The option price is computed by evaluating the integral transform given by Lewis (2000) using the form of the characteristic function discussed by Albrecher *et al.* (2007), see also Kilin (2006).

$$P_{\text{call}} = Se^{-qT} - Xe^{-rT} \frac{1}{\pi} \text{Re} \left[ \int_{0+i/2}^{\infty+i/2} e^{-ik\bar{X}} \frac{\hat{H}(k, v, T)}{k^2 - ik} dk \right], \quad (1)$$

where  $\bar{X} = \ln(S/X) + (r - q)T$  and

$$\hat{H}(k, v, T) = \exp\left(\frac{2\kappa\eta}{\sigma_v^2} \left[ \text{tgendgroup} - \ln\left(\frac{1 - h e^{-\xi t}}{1 - h}\right) \right] + v_{\text{tg}} \left[ \frac{1 - e^{-\xi t}}{1 - h e^{-\xi t}} \right] \right),$$

$$g = \frac{1}{2}(b - \xi), \quad h = \frac{b - \xi}{b + \xi}, \quad t = \sigma_v^2 T / 2,$$

$$\xi = \left[ b^2 + 4 \frac{k^2 - ik}{\sigma_v^2} \right]^{\frac{1}{2}},$$

$$b = \frac{2}{\sigma_v^2} \left[ (1 - \gamma + ik) \rho \sigma_v + \sqrt{\kappa^2 - \gamma(1 - \gamma) \sigma_v^2} \right]$$

with  $t = \sigma_v^2 T / 2$ . Here  $\gamma$  is the risk aversion parameter of the representative agent with  $0 \leq \gamma \leq 1$  and  $\gamma(1 - \gamma) \sigma_v^2 \leq \kappa^2$ . The value  $\gamma = 1$  corresponds to  $\lambda = 0$ , where  $\lambda$  is the market price of risk in Heston (1993) (see Lewis (2000) and Rouah and Vainberg (2007)).

The price of a put option is obtained by put-call parity.

$$P_{\text{put}} = P_{\text{call}} + X e^{-rT} - S e^{-qT}.$$

Writing the expression for the price of a call option as

$$P_{\text{call}} = S e^{-qT} - X e^{-rT} \frac{1}{\pi} \operatorname{Re} \left[ \int_{0+i/2}^{\infty+i/2} I(k, r, S, T, v) dk \right]$$

then the sensitivities or Greeks can be obtained in the following manner,

Delta

$$\frac{\partial P_{\text{call}}}{\partial S} = e^{-qT} + \frac{X e^{-rT}}{S} \frac{1}{\pi} \operatorname{Re} \left[ \int_{0+i/2}^{\infty+i/2} (ik) I(k, r, S, T, v) dk \right],$$

Vega

$$\frac{\partial P}{\partial v} = -X e^{-rT} \frac{1}{\pi} \operatorname{Re} \left[ \int_{0-i/2}^{0+i/2} f_2 I(k, r, j, S, T, v) dk \right], \quad \text{where } f_2 = g \left[ \frac{1 - e^{-\xi t}}{1 - h e^{-\xi t}} \right],$$

Rho

$$\frac{\partial P_{\text{call}}}{\partial r} = T X e^{-rT} \frac{1}{\pi} \operatorname{Re} \left[ \int_{0+i/2}^{\infty+i/2} (1 + ik) I(k, r, S, T, v) dk \right].$$

The option price  $P_{ij} = P(X = X_i, T = T_j)$  is computed for each strike price in a set  $X_i$ ,  $i = 1, 2, \dots, m$ , and for each expiry time in a set  $T_j$ ,  $j = 1, 2, \dots, n$ .

## 4 References

Albrecher H, Mayer P, Schoutens W and Tistaert J (2007) The little Heston trap *Wilmott Magazine* **January 2007** 83–92

Heston S (1993) A closed-form solution for options with stochastic volatility with applications to bond and currency options *Review of Financial Studies* **6** 327–343

Kilin F (2006) Accelerating the calibration of stochastic volatility models *MPRA Paper No. 2975* <http://mpra.ub.uni-muenchen.de/2975/>

Lewis A L (2000) Option valuation under stochastic volatility *Finance Press, USA*

Rouah F D and Vainberg G (2007) *Option Pricing Models and Volatility using Excel-VBA* John Wiley and Sons, Inc

## 5 Parameters

- 1: CALPUT – CHARACTER(1) *Input*  
*On entry:* determines whether the option is a call or a put.  
 CALPUT = 'C'  
 A call; the holder has a right to buy.  
 CALPUT = 'P'  
 A put; the holder has a right to sell.  
*Constraint:* CALPUT = 'C' or 'P'.
- 2: M – INTEGER *Input*  
*On entry:* the number of strike prices to be used.  
*Constraint:*  $M \geq 1$ .
- 3: N – INTEGER *Input*  
*On entry:* the number of times to expiry to be used.  
*Constraint:*  $N \geq 1$ .
- 4: X(M) – REAL (KIND=nag\_wp) array *Input*  
*On entry:* X(*i*) must contain  $X_i$ , the *i*th strike price, for  $i = 1, 2, \dots, M$ .  
*Constraint:*  $X(i) \geq z$  and  $X(i) \leq 1/z$ , where  $z = X02AMF()$ , the safe range parameter, for  $i = 1, 2, \dots, M$ .
- 5: S – REAL (KIND=nag\_wp) *Input*  
*On entry:*  $S$ , the price of the underlying asset.  
*Constraint:*  $S \geq z$  and  $S \leq 1.0/z$ , where  $z = X02AMF()$ , the safe range parameter.
- 6: T(N) – REAL (KIND=nag\_wp) array *Input*  
*On entry:* T(*i*) must contain  $T_i$ , the *i*th time, in years, to expiry, for  $i = 1, 2, \dots, N$ .  
*Constraint:*  $T(i) \geq z$ , where  $z = X02AMF()$ , the safe range parameter, for  $i = 1, 2, \dots, N$ .
- 7: SIGMAV – REAL (KIND=nag\_wp) *Input*  
*On entry:* the volatility,  $\sigma_v$ , of the volatility process,  $\sqrt{v_t}$ . Note that a rate of 20% should be entered as 0.2.  
*Constraint:* SIGMAV > 0.0.
- 8: KAPPA – REAL (KIND=nag\_wp) *Input*  
*On entry:*  $\kappa$ , the long term mean reversion rate of the volatility.  
*Constraint:* KAPPA > 0.0.
- 9: CORR – REAL (KIND=nag\_wp) *Input*  
*On entry:* the correlation between the two standard Brownian motions for the asset price and the volatility.  
*Constraint:*  $-1.0 \leq CORR \leq 1.0$ .

- 10: VAR0 – REAL (KIND=nag\_wp) Input  
*On entry:* the initial value of the variance,  $v_t$ , of the asset price.  
*Constraint:* VAR0  $\geq$  0.0.
- 11: ETA – REAL (KIND=nag\_wp) Input  
*On entry:*  $\eta$ , the long term mean of the variance of the asset price.  
*Constraint:* ETA  $>$  0.0.
- 12: GRISK – REAL (KIND=nag\_wp) Input  
*On entry:* the risk aversion parameter,  $\gamma$ , of the representative agent.  
*C o n s t r a i n t :*  $0.0 \leq \text{GRISK} \leq 1.0$  a n d  
 GRISK  $\times$  (1 – GRISK)  $\times$  SIGMAV  $\times$  SIGMAV  $\leq$  KAPPA  $\times$  KAPPA.
- 13: R – REAL (KIND=nag\_wp) Input  
*On entry:*  $r$ , the annual risk-free interest rate, continuously compounded. Note that a rate of 5% should be entered as 0.05.  
*Constraint:* R  $\geq$  0.0.
- 14: Q – REAL (KIND=nag\_wp) Input  
*On entry:*  $q$ , the annual continuous yield rate. Note that a rate of 8% should be entered as 0.08.  
*Constraint:* Q  $\geq$  0.0.
- 15: P(LDP, N) – REAL (KIND=nag\_wp) array Output  
*On exit:* P( $i, j$ ) contains  $P_{ij}$ , the option price evaluated for the strike price  $X_i$  at expiry  $T_j$  for  $i = 1, 2, \dots, M$  and  $j = 1, 2, \dots, N$ .
- 16: LDP – INTEGER Input  
*On entry:* the first dimension of the arrays P, DELTA, GAMMA, VEGA, THETA, RHO, VANNA, CHARM, SPEED, ZOMMA and VOMMA as declared in the (sub)program from which S30NBF is called.  
*Constraint:* LDP  $\geq$  M.
- 17: DELTA(LDP, N) – REAL (KIND=nag\_wp) array Output  
*On exit:* the leading  $M \times N$  part of the array DELTA contains the sensitivity,  $\frac{\partial P}{\partial S}$ , of the option price to change in the price of the underlying asset.
- 18: GAMMA(LDP, N) – REAL (KIND=nag\_wp) array Output  
*On exit:* the leading  $M \times N$  part of the array GAMMA contains the sensitivity,  $\frac{\partial^2 P}{\partial S^2}$ , of DELTA to change in the price of the underlying asset.
- 19: VEGA(LDP, N) – REAL (KIND=nag\_wp) array Output  
*On exit:* VEGA( $i, j$ ), contains the first-order Greek measuring the sensitivity of the option price  $P_{ij}$  to change in the volatility of the underlying asset, i.e.,  $\frac{\partial P_{ij}}{\partial \sigma}$ , for  $i = 1, 2, \dots, M$  and  $j = 1, 2, \dots, N$ .
- 20: THETA(LDP, N) – REAL (KIND=nag\_wp) array Output  
*On exit:* THETA( $i, j$ ), contains the first-order Greek measuring the sensitivity of the option price  $P_{ij}$  to change in time, i.e.,  $-\frac{\partial P_{ij}}{\partial T}$ , for  $i = 1, 2, \dots, M$  and  $j = 1, 2, \dots, N$ , where  $b = r - q$ .

- 21: RHO(LDP, N) – REAL (KIND=nag\_wp) array *Output*  
*On exit:* RHO( $i, j$ ), contains the first-order Greek measuring the sensitivity of the option price  $P_{ij}$  to change in the annual risk-free interest rate, i.e.,  $-\frac{\partial P_{ij}}{\partial r}$ , for  $i = 1, 2, \dots, M$  and  $j = 1, 2, \dots, N$ .
- 22: VANNA(LDP, N) – REAL (KIND=nag\_wp) array *Output*  
*On exit:* VANNA( $i, j$ ), contains the second-order Greek measuring the sensitivity of the first-order Greek  $\Delta_{ij}$  to change in the volatility of the asset price, i.e.,  $-\frac{\partial \Delta_{ij}}{\partial T} = -\frac{\partial^2 P_{ij}}{\partial S \partial \sigma}$ , for  $i = 1, 2, \dots, M$  and  $j = 1, 2, \dots, N$ .
- 23: CHARM(LDP, N) – REAL (KIND=nag\_wp) array *Output*  
*On exit:* CHARM( $i, j$ ), contains the second-order Greek measuring the sensitivity of the first-order Greek  $\Delta_{ij}$  to change in the time, i.e.,  $-\frac{\partial \Delta_{ij}}{\partial T} = -\frac{\partial^2 P_{ij}}{\partial S \partial T}$ , for  $i = 1, 2, \dots, M$  and  $j = 1, 2, \dots, N$ .
- 24: SPEED(LDP, N) – REAL (KIND=nag\_wp) array *Output*  
*On exit:* SPEED( $i, j$ ), contains the third-order Greek measuring the sensitivity of the second-order Greek  $\Gamma_{ij}$  to change in the price of the underlying asset, i.e.,  $-\frac{\partial \Gamma_{ij}}{\partial S} = -\frac{\partial^3 P_{ij}}{\partial S^3}$ , for  $i = 1, 2, \dots, M$  and  $j = 1, 2, \dots, N$ .
- 25: ZOMMA(LDP, N) – REAL (KIND=nag\_wp) array *Output*  
*On exit:* ZOMMA( $i, j$ ), contains the third-order Greek measuring the sensitivity of the second-order Greek  $\Gamma_{ij}$  to change in the volatility of the underlying asset, i.e.,  $-\frac{\partial \Gamma_{ij}}{\partial \sigma} = -\frac{\partial^3 P_{ij}}{\partial S^2 \partial \sigma}$ , for  $i = 1, 2, \dots, M$  and  $j = 1, 2, \dots, N$ .
- 26: VOMMA(LDP, N) – REAL (KIND=nag\_wp) array *Output*  
*On exit:* VOMMA( $i, j$ ), contains the second-order Greek measuring the sensitivity of the first-order Greek  $\Delta_{ij}$  to change in the volatility of the underlying asset, i.e.,  $-\frac{\partial \Delta_{ij}}{\partial \sigma} = -\frac{\partial^2 P_{ij}}{\partial \sigma^2}$ , for  $i = 1, 2, \dots, M$  and  $j = 1, 2, \dots, N$ .
- 27: IFAIL – INTEGER *Input/Output*  
*On entry:* IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.  
 For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**  
*On exit:* IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

## 6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, CALPUT =  $\langle value \rangle$  was an illegal value.

IFAIL = 2

On entry,  $M = \langle value \rangle$ .  
Constraint:  $M \geq 1$ .

IFAIL = 3

On entry,  $N = \langle value \rangle$ .  
Constraint:  $N \geq 1$ .

IFAIL = 4

On entry,  $X(\langle value \rangle) = \langle value \rangle$ .  
Constraint:  $X(i) \geq \langle value \rangle$  and  $X(i) \leq \langle value \rangle$ .

IFAIL = 5

On entry,  $S = \langle value \rangle$ .  
Constraint:  $S \geq \langle value \rangle$  and  $S \leq \langle value \rangle$ .

IFAIL = 6

On entry,  $T(\langle value \rangle) = \langle value \rangle$ .  
Constraint:  $T(i) \geq \langle value \rangle$ .

IFAIL = 7

On entry,  $SIGMAV = \langle value \rangle$ .  
Constraint:  $SIGMAV > 0.0$ .

IFAIL = 8

On entry,  $KAPPA = \langle value \rangle$ .  
Constraint:  $KAPPA > 0.0$ .

IFAIL = 9

On entry,  $CORR = \langle value \rangle$ .  
Constraint:  $|CORR| \leq 1.0$ .

IFAIL = 10

On entry,  $VAR0 = \langle value \rangle$ .  
Constraint:  $VAR0 \geq 0.0$ .

IFAIL = 11

On entry,  $ETA = \langle value \rangle$ .  
Constraint:  $ETA > 0.0$ .

IFAIL = 12

On entry,  $GRISK = \langle value \rangle$ ,  $SIGMAV = \langle value \rangle$  and  $KAPPA = \langle value \rangle$ .  
Constraint:  $0.0 \leq GRISK \leq 1.0$  and  $GRISK \times (1.0 - GRISK) \times SIGMAV^2 \leq KAPPA^2$ .

IFAIL = 13

On entry,  $R = \langle value \rangle$ .  
Constraint:  $R \geq 0.0$ .

IFAIL = 14

On entry,  $Q = \langle value \rangle$ .  
Constraint:  $Q \geq 0.0$ .

IFAIL = 16

On entry, LDP =  $\langle value \rangle$  and M =  $\langle value \rangle$ .  
Constraint: LDP  $\geq$  M.

IFAIL = 17

Quadrature has not converged to the required accuracy. However, the result should be a reasonable approximation.

IFAIL = 18

Solution cannot be computed accurately. Check values of input parameters.

IFAIL = -99

An unexpected error has been triggered by this routine. Please contact NAG.  
See Section 3.8 in the Essential Introduction for further information.

IFAIL = -399

Your licence key may have expired or may not have been installed correctly.  
See Section 3.7 in the Essential Introduction for further information.

IFAIL = -999

Dynamic memory allocation failed.  
See Section 3.6 in the Essential Introduction for further information.

## 7 Accuracy

The accuracy of the output is determined by the accuracy of the numerical quadrature used to evaluate the integral in (1). An adaptive method is used which evaluates the integral to within a tolerance of  $\max(10^{-8}, 10^{-10} \times |I|)$ , where  $|I|$  is the absolute value of the integral.

## 8 Parallelism and Performance

S30NBF is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

None.

## 10 Example

This example computes the price and sensitivities of a European call using Heston's stochastic volatility model. The time to expiry is 1 year, the stock price is 100 and the strike price is 100. The risk-free interest rate is 2.5% per year, the volatility of the variance,  $\sigma_v$ , is 57.51% per year, the mean reversion parameter,  $\kappa$ , is 1.5768, the long term mean of the variance,  $\eta$ , is 0.0398 and the correlation between the volatility process and the stock price process,  $\rho$ , is -0.5711. The risk aversion parameter,  $\gamma$ , is 1.0 and the initial value of the variance, VAR0, is 0.0175.

## 10.1 Program Text

```

Program s30nbfe

!      S30NBF Example Program Text

!      Mark 25 Release. NAG Copyright 2014.

!      .. Use Statements ..
Use nag_library, Only: nag_wp, s30nbfe
!      .. Implicit None Statement ..
Implicit None
!      .. Parameters ..
Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
Real (Kind=nag_wp)         :: corr, eta, grisk, kappa, q, r, s,      &
                             sigmav, var0
Integer                    :: i, ifail, j, ldp, m, n
Character (1)              :: calput
!      .. Local Arrays ..
Real (Kind=nag_wp), Allocatable :: charm(:,,:), delta(:,,:), gamma(:,,:), &
                             p(:,,:), rho(:,,:), speed(:,,:), t(:,) &
                             theta(:,,:), vanna(:,,:), vega(:,,:), &
                             vomma(:,,:), x(:,), zomma(:,)
!      .. Executable Statements ..
Write (nout,*) 'S30NBF Example Program Results'

!      Skip heading in data file
Read (nin,*)

Read (nin,*) calput
Read (nin,*) s, r, q
Read (nin,*) kappa, eta, var0, sigmav, corr, grisk
Read (nin,*) m, n

ldp = m
Allocate (charm(ldp,n),delta(ldp,n),gamma(ldp,n),p(ldp,n),rho(ldp,n), &
          speed(ldp,n),t(n),theta(ldp,n),vanna(ldp,n),vega(ldp,n),vomma(ldp,n), &
          x(m),zomma(ldp,n))

Read (nin,*)(x(i),i=1,m)
Read (nin,*)(t(i),i=1,n)

ifail = 0
Call s30nbfe(calput,m,n,x,s,t,sigmav,kappa,corr,var0,eta,grisk,r,q,p,ldp, &
             delta,gamma,vega,theta,rho,vanna,charm,speed,zomma,vomma,ifail)

Write (nout,*)
Write (nout,*) 'Heston''s Stochastic volatility Model'

Select Case (calput)
Case ('C','c')
  Write (nout,*) 'European Call :'
Case ('P','p')
  Write (nout,*) 'European Put :'
End Select

Write (nout,99997) ' Spot = ', s
Write (nout,99997) ' Volatility of vol = ', sigmav
Write (nout,99997) ' Mean reversion = ', kappa
Write (nout,99997) ' Correlation = ', corr
Write (nout,99997) ' Variance = ', var0
Write (nout,99997) ' Mean of variance = ', eta
Write (nout,99997) ' Risk aversion = ', grisk
Write (nout,99997) ' Rate = ', r
Write (nout,99997) ' Dividend = ', q

Write (nout,*)

Do j = 1, n
  Write (nout,*)

```



```

Write (nout,99999) t(j)
Write (nout,*) &
  ' Strike      Price      Delta      Gamma      Vega      ' // &
  'Theta      Rho'

Do i = 1, m
  Write (nout,99998) x(i), p(i,j), delta(i,j), gamma(i,j), vega(i,j), &
    theta(i,j), rho(i,j)
End Do

Write (nout,*) &
  ' Strike      Price      Vanna      Charm      Speed      ' // &
  'Zomma      Vomma'

Do i = 1, m
  Write (nout,99998) x(i), p(i,j), vanna(i,j), charm(i,j), speed(i,j), &
    zomma(i,j), vomma(i,j)
End Do

End Do

99999 Format (1X,'Time to Expiry : ',1X,F8.4)
99998 Format (1X,7(F10.4,1X))
99997 Format (A,1X,F10.4)
End Program s30nbfe

```

## 10.2 Program Data

S30NBF Example Program Data

```

'C' : Call = 'C', Put = 'P'
100.0 0.025 0.0 : S, R, Q
1.5768 0.0398 0.0175 0.5751 -0.5711 1.0 : KAPPA, ETA, VARO, SIGMAV, CORR, GRISK
1 1 : M, N
100.0 : X(I), I = 1,2,...N
1.0 : T(I), I = 1,2,...M

```

## 10.3 Program Results

S30NBF Example Program Results

Heston's Stochastic volatility Model

European Call :

```

Spot = 100.0000
Volatility of vol = 0.5751
Mean reversion = 1.5768
Correlation = -0.5711
Variance = 0.0175
Mean of variance = 0.0398
Risk aversion = 1.0000
Rate = 0.0250
Dividend = 0.0000

```

Time to Expiry : 1.0000

Strike	Price	Delta	Gamma	Vega	Theta	Rho
100.0000	7.2743	0.6945	0.0251	52.5461	-4.9969	62.1735
Strike	Price	Vanna	Charm	Speed	Zomma	Vomma
100.0000	7.2743	-0.5643	-0.0321	-0.0023	-0.1976	-321.0780