NAG Library Routine Document

S19ANF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S19ANF returns an array of values for the Kelvin function $\operatorname{ber} x$.

2 Specification

```
SUBROUTINE S19ANF (N, X, F, IVALID, IFAIL)
INTEGER            N, IVALID(N), IFAIL
REAL (KIND=nag_wp) X(N), F(N)
```

3 Description

S19ANF evaluates an approximation to the Kelvin function $\operatorname{ber} x_i$ for an array of arguments x_i, for $i = 1, 2, \ldots, n$.

Note: $\operatorname{ber}(-x) = \operatorname{ber} x$, so the approximation need only consider $x \geq 0.0$.

The routine is based on several Chebyshev expansions:

For $0 \leq x \leq 5$,

$$\operatorname{ber} x = \sum_{r=0} a_r T_r(t), \qquad \text{with } t = 2\left(\frac{x}{5}\right)^4 - 1.$$

For $x > 5$,

$$\operatorname{ber} x = \frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}}\left[\left(1 + \frac{1}{x}a(t)\right)\cos\alpha + \frac{1}{x}b(t)\sin\alpha\right]$$

$$+\frac{e^{-x/\sqrt{2}}}{\sqrt{2\pi x}}\left[\left(1 + \frac{1}{x}c(t)\right)\sin\beta + \frac{1}{x}d(t)\cos\beta\right],$$

where $\alpha = \frac{x}{\sqrt{2}} - \frac{\pi}{8}$, $\beta = \frac{x}{\sqrt{2}} + \frac{\pi}{8}$,

and $a(t)$, $b(t)$, $c(t)$, and $d(t)$ are expansions in the variable $t = \frac{10}{x} - 1$.

When x is sufficiently close to zero, the result is set directly to $\operatorname{ber} 0 = 1.0$.

For large x, there is a danger of the result being totally inaccurate, as the error amplification factor grows in an essentially exponential manner; therefore the routine must fail.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: N – INTEGER *Input*

On entry: n, the number of points.

Constraint: $N \geq 0$.

2: X(N) – REAL (KIND=nag_wp) array *Input*

On entry: the argument x_i of the function, for $i = 1, 2, \ldots, N$.

3: F(N) – REAL (KIND=nag_wp) array *Output*

On exit: $\operatorname{ber} x_i$, the function values.

4: IVALID(N) – INTEGER array *Output*

On exit: IVALID(i) contains the error code for x_i, for $i = 1, 2, \ldots, N$.

IVALID$(i) = 0$
No error.

IVALID$(i) = 1$
abs(x_i) is too large for an accurate result to be returned. F(i) contains zero. The threshold value is the same as for IFAIL $= 1$ in S19AAF, as defined in the Users' Note for your implementation.

5: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, at least one value of X was invalid.
Check IVALID for more information.

IFAIL $= 2$

On entry, N $= \langle$*value*$\rangle$.
Constraint: $N \geq 0$.

IFAIL $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.8 in the Essential Introduction for further information.

IFAIL $= -399$

Your licence key may have expired or may not have been installed correctly.

See Section 3.7 in the Essential Introduction for further information.

IFAIL $= -999$

Dynamic memory allocation failed.

See Section 3.6 in the Essential Introduction for further information.

7 Accuracy

Since the function is oscillatory, the absolute error rather than the relative error is important. Let E be the absolute error in the result and δ be the relative error in the argument. If δ is somewhat larger than the ***machine precision***, then we have:

$$E \simeq \left|\frac{x}{\sqrt{2}}(\mathrm{ber}_1\, x + \mathrm{bei}_1\, x)\right|\delta$$

(provided E is within machine bounds).

For small x the error amplification is insignificant and thus the absolute error is effectively bounded by the ***machine precision***.

For medium and large x, the error behaviour is oscillatory and its amplitude grows like $\sqrt{\frac{x}{2\pi}}e^{x/\sqrt{2}}$. Therefore it is not possible to calculate the function with any accuracy when $\sqrt{x}e^{x/\sqrt{2}} > \frac{\sqrt{2\pi}}{\delta}$. Note that this value of x is much smaller than the minimum value of x for which the function overflows.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of X from a file, evaluates the function at each value of x_i and prints the results.

10.1 Program Text

```
    Program s19anfe

!     S19ANF Example Program Text

!     Mark 25 Release. NAG Copyright 2014.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, s19anf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter                :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                           :: i, ifail, n
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable   :: f(:), x(:)
      Integer, Allocatable              :: ivalid(:)
```

```
!     .. Executable Statements ..
      Write (nout,*) 'S19ANF Example Program Results'

!     Skip heading in data file
      Read (nin,*)

      Write (nout,*)
      Write (nout,*) '     X           F          IVALID'
      Write (nout,*)

      Read (nin,*) n

      Allocate (x(n),f(n),ivalid(n))

      Read (nin,*) x(1:n)

      ifail = 0
      Call s19anf(n,x,f,ivalid,ifail)

      Do i = 1, n
        Write (nout,99999) x(i), f(i), ivalid(i)
      End Do

99999 Format (1X,1P,2E12.3,I5)
    End Program s19anfe
```

10.2 Program Data

```
S19ANF Example Program Data

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0.1 1.0 2.5 5.0 10.0 15.0 -1.0
```

10.3 Program Results

```
 S19ANF Example Program Results

      X           F          IVALID

    1.000E-01   1.000E+00    0
    1.000E+00   9.844E-01    0
    2.500E+00   4.000E-01    0
    5.000E+00  -6.230E+00    0
    1.000E+01   1.388E+02    0
    1.500E+01  -2.967E+03    0
   -1.000E+00   9.844E-01    0
```