NAG Library Routine Document

S14CBF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S14CBF returns the value of the logarithm of the beta function, $\ln B(a,b)$, via the routine name.

2 Specification

```
FUNCTION S14CBF (A, B, IFAIL)
REAL (KIND=nag_wp) S14CBF
INTEGER            IFAIL
REAL (KIND=nag_wp) A, B
```

3 Description

S14CBF calculates values for $\ln B(a,b)$ where B is the beta function given by

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}\,dt$$

or equivalently

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

and $\Gamma(x)$ is the gamma function. Note that the beta function is symmetric, so that $B(a,b) = B(b,a)$.

In order to efficiently obtain accurate results several methods are used depending on the parameters a and b.

Let $a_0 = \min(a,b)$ and $b_0 = \max(a,b)$. Then:

for $a_0 \geq 8$,

$$\ln B \quad = \quad 0.5\ln(2\pi) - 0.5\ln(b_0) + \Delta(a_0) + \Delta(b_0) - \Delta(a_0+b_0) - u - v;$$

where

$\Delta(a_0) = \ln\Gamma(a_0) - (a_0 - 0.5)\ln a_0 + a_0 - 0.5\ln(2\pi)$,

$u = -(a_0 - 0.5)\ln\left[\frac{a_0}{a_0+b_0}\right]$ and

$v = b_0 \ln\left(1 + \frac{a_0}{b_0}\right)$.

for $a_0 < 1$,

for $b_0 \geq 8$,

$$\ln B = \ln\Gamma(a_0) + \ln\frac{\Gamma(b_0)}{\Gamma(a_0+b_0)};$$

for $b_0 < 8$,

$$\ln B = \ln\Gamma(a_0) + \ln\Gamma(b_0) - \ln\Gamma(a_0+b_0);$$

for $2 < a_0 < 8$, a_0 is reduced to the interval $[1,2]$ by $B(a,b) = \frac{a_0-1}{a_0+b_0-1}B(a_0-1,b_0)$;

for $1 \leq a_0 \leq 2$,

for $b_0 \geq 8$,

$$\ln B = \ln \Gamma(a_0) + \ln \frac{\Gamma(b_0)}{\Gamma(a_0 + b_0)};$$

for $2 < b_0 < 8$, b_0 is reduced to the interval $[1, 2]$;

for $b_0 \leq 2$,

$$\ln B = \ln \Gamma(a_0) + \ln \Gamma(b_0) - \ln \Gamma(a_0 + b_0).$$

S14CBF is derived from BETALN in DiDonato and Morris (1992).

4 References

DiDonato A R and Morris A H (1992) Algorithm 708: Significant digit computation of the incomplete beta function ratios *ACM Trans. Math. Software* **18** 360–373

5 Parameters

1: A – REAL (KIND=nag_wp) *Input*

On entry: the argument a of the function.

Constraint: A > 0.0.

2: B – REAL (KIND=nag_wp) *Input*

On entry: the argument b of the function.

Constraint: B > 0.0.

3: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, A $= \langle value \rangle$.
Constraint: A > 0.0.

On entry, B $= \langle value \rangle$.
Constraint: B > 0.0.

IFAIL $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.8 in the Essential Introduction for further information.

IFAIL $= -399$

Your licence key may have expired or may not have been installed correctly.

See Section 3.7 in the Essential Introduction for further information.

IFAIL $= -999$

Dynamic memory allocation failed.

See Section 3.6 in the Essential Introduction for further information.

7 Accuracy

S14CBF should produce full relative accuracy for all input arguments.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of the arguments a and b from a file, evaluates the function and prints the results.

10.1 Program Text

```
    Program s14cbfe

!     S14CBF Example Program Text
!     Mark 25 Release. NAG Copyright 2014.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, s14cbf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: a, b, lb
      Integer                          :: i, ifail
!     .. Executable Statements ..
      Write (nout,*) 'S14CBF Example Program Results'
      Write (nout,*)
      Write (nout,*) ' A    B        ln(beta(A,B))'
      Write (nout,*)

!     Skip heading in data file
      Read (nin,*)

      Do
        Read (nin,*,Iostat=i) a, b
        If (i/=0) Exit

        ifail = -1
        lb = s14cbf(a,b,ifail)
```

```
        If (ifail==0) Then
          Write (nout,99999) a, b, lb
        End If
      End Do

99999 Format (2F5.2,1P,E17.4)
    End Program s14cbfe
```

10.2 Program Data

```
S14CBF Example Program Data
0.2   1.0
0.4   1.0
0.6   1.0
0.8   1.0
1.0   0.2
1.0   0.4
1.0   1.0
2.0   2.0
3.0   3.0
4.0   4.0
5.0   5.0
6.0   2.0
6.0   3.0
6.0   4.0
6.0   5.0
6.0   6.0
7.0   7.0       : A, B
```

10.3 Program Results

```
 S14CBF Example Program Results

  A    B        ln(beta(A,B))

 0.20 1.00       1.6094E+00
 0.40 1.00       9.1629E-01
 0.60 1.00       5.1083E-01
 0.80 1.00       2.2314E-01
 1.00 0.20       1.6094E+00
 1.00 0.40       9.1629E-01
 1.00 1.00       0.0000E+00
 2.00 2.00      -1.7918E+00
 3.00 3.00      -3.4012E+00
 4.00 4.00      -4.9416E+00
 5.00 5.00      -6.4457E+00
 6.00 2.00      -3.7377E+00
 6.00 3.00      -5.1240E+00
 6.00 4.00      -6.2226E+00
 6.00 5.00      -7.1389E+00
 6.00 6.00      -7.9273E+00
 7.00 7.00      -9.3937E+00
```