# NAG Library Routine Document

# S13ACF

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

## 1 Purpose

S13ACF returns the value of the cosine integral

$$\mathrm{Ci}(x) = \gamma + \ln x + \int_0^x \frac{\cos u - 1}{u}\, du, \quad x > 0$$

via the routine name where $\gamma$ denotes Euler's constant.

## 2 Specification

```
FUNCTION S13ACF (X, IFAIL)
REAL (KIND=nag_wp) S13ACF

INTEGER            IFAIL
REAL (KIND=nag_wp) X
```

## 3 Description

S13ACF calculates an approximate value for $\mathrm{Ci}(x)$.

For $0 < x \leq 16$ it is based on the Chebyshev expansion

$$\mathrm{Ci}(x) = \ln x + \sum_{r=0}{}' a_r T_r(t), t = 2\left(\frac{x}{16}\right)^2 - 1.$$

For $16 < x < x_{\mathrm{hi}}$ where the value of $x_{\mathrm{hi}}$ is given in the Users' Note for your implementation,

$$\mathrm{Ci}(x) = \frac{f(x)\sin x}{x} - \frac{g(x)\cos x}{x^2}$$

where $f(x) = \sum_{r=0} f_r T_r(t)$ and $g(x) = \sum_{r=0} g_r T_r(t)$, $t = 2\left(\frac{16}{x}\right)^2 - 1$.

For $x \geq x_{\mathrm{hi}}$, $\mathrm{Ci}(x) = 0$ to within the accuracy possible (see Section 7).

## 4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

## 5 Parameters

1: X – REAL (KIND=nag_wp) *Input*

*On entry*: the argument $x$ of the function.

*Constraint*: $\mathrm{X} > 0.0$.

2: IFAIL – INTEGER *Input/Output*

*On entry*: IFAIL must be set to 0, $-1$ or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value $-1$ or $1$ is recommended. If the output of error messages is undesirable, then the value $1$ is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is $0$. **When the value $-1$ or $1$ is used it is essential to test the value of IFAIL on exit.**

*On exit*: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

## 6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or $-1$, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

The routine has been called with an argument less than or equal to zero for which the function is not defined. The result returned is zero.

IFAIL $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.8 in the Essential Introduction for further information.

IFAIL $= -399$

Your licence key may have expired or may not have been installed correctly.

See Section 3.7 in the Essential Introduction for further information.

IFAIL $= -999$

Dynamic memory allocation failed.

See Section 3.6 in the Essential Introduction for further information.

## 7 Accuracy

If $E$ and $\epsilon$ are the absolute and relative errors in the result and $\delta$ is the relative error in the argument then in principle these are related by

$$|E| \simeq |\delta \cos x| \text{and } |\epsilon| \simeq \left|\frac{\delta \cos x}{\mathrm{Ci}(x)}\right|.$$

That is accuracy will be limited by ***machine precision*** near the origin and near the zeros of $\cos x$, but near the zeros of $\mathrm{Ci}(x)$ only absolute accuracy can be maintained.

The behaviour of this amplification is shown in Figure 1.

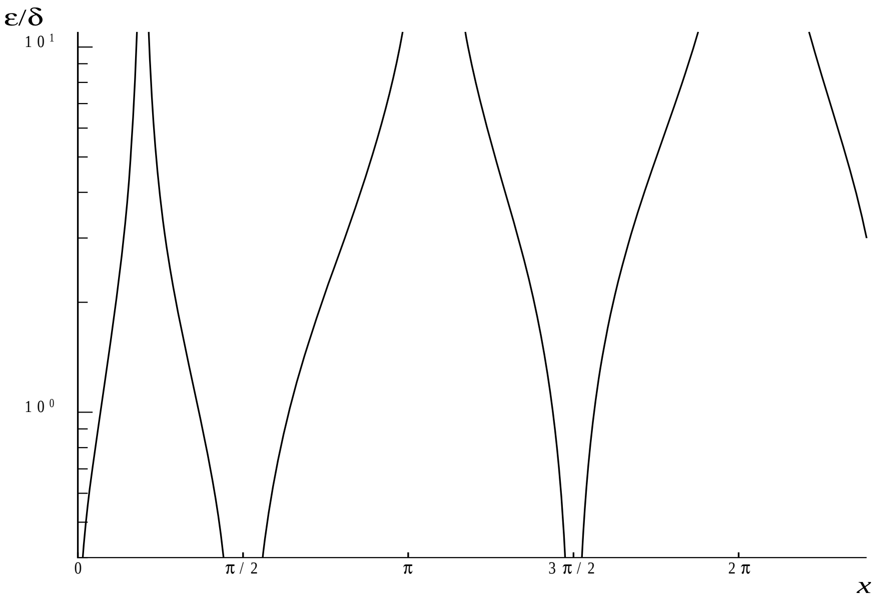



**Figure 1**

For large values of $x$, $\mathrm{Ci}(x) \sim \frac{\sin x}{x}$ therefore $\epsilon \sim \delta x \cot x$ and since $\delta$ is limited by the finite precision of the machine it becomes impossible to return results which have any relative accuracy. That is, when $x \geq 1/\delta$ we have that $|\mathrm{Ci}(x)| \leq 1/x \sim E$ and hence is not significantly different from zero.

Hence $x_{\mathrm{hi}}$ is chosen such that for values of $x \geq x_{\mathrm{hi}}$, $\mathrm{Ci}(x)$ in principle would have values less than the ***machine precision*** and so is essentially zero.

## 8 Parallelism and Performance

Not applicable.

## 9 Further Comments

None.

## 10 Example

This example reads values of the argument $x$ from a file, evaluates the function at each value of $x$ and prints the results.

### 10.1 Program Text

```
    Program s13acfe

!     S13ACF Example Program Text

!     Mark 25 Release. NAG Copyright 2014.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, s13acf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: x, y
      Integer                          :: ifail, ioerr
!     .. Executable Statements ..
      Write (nout,*) 'S13ACF Example Program Results'
```

```
!     Skip heading in data file
      Read (nin,*)

      Write (nout,*)
      Write (nout,*) '     X            Y'
      Write (nout,*)

data: Do
        Read (nin,*,Iostat=ioerr) x

        If (ioerr<0) Then
          Exit data
        End If

        ifail = -1
        y = s13acf(x,ifail)

        If (ifail<0) Then
          Exit data
        End If

        Write (nout,99999) x, y
      End Do data

99999 Format (1X,1P,2E12.3)
    End Program s13acfe
```

## 10.2 Program Data

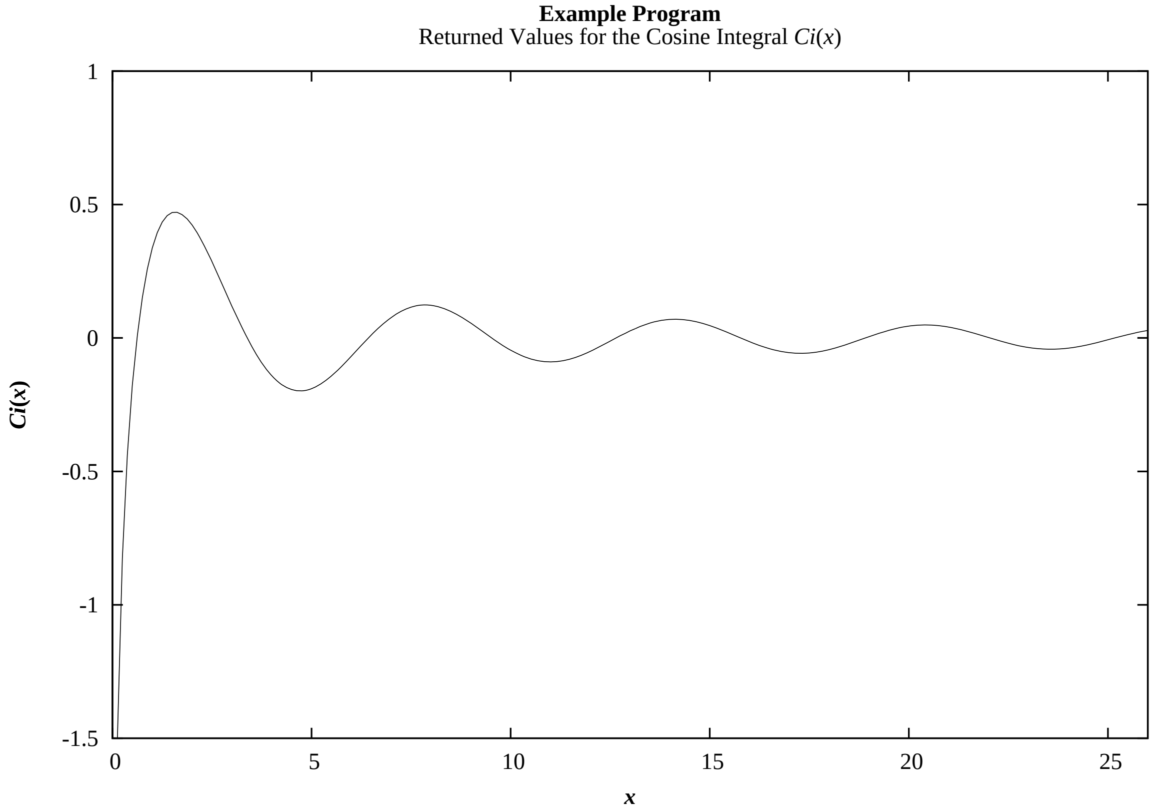
```
S13ACF Example Program Data
             0.2
             0.4
             0.6
             0.8
             1.0
```

## 10.3 Program Results

```
 S13ACF Example Program Results

      X           Y

    2.000E-01  -1.042E+00
    4.000E-01  -3.788E-01
    6.000E-01  -2.227E-02
    8.000E-01   1.983E-01
    1.000E+00   3.374E-01
```


Example Program
Returned Values for the Cosine Integral Ci(x)
1
0.5
0
-0.5
-1
-1.5
0
5
10
15
20
25
Ci(x)
x