NAG Library Routine Document

S11AAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S11AAF returns the value of the inverse hyperbolic tangent, $\operatorname{arctanh} x$, via the function name.

2 Specification

```
FUNCTION S11AAF (X, IFAIL)
REAL (KIND=nag_wp) S11AAF

INTEGER          IFAIL
REAL (KIND=nag_wp) X
```

3 Description

S11AAF calculates an approximate value for the inverse hyperbolic tangent of its argument, $\operatorname{arctanh} x$.

For $x^2 \leq \frac{1}{2}$ it is based on the Chebyshev expansion

$$\operatorname{arctanh} x = x \times y(t) = x \sum_{r=0} a_r T_r(t)$$

where $-\dfrac{1}{\sqrt{2}} \leq x \leq \dfrac{1}{\sqrt{2}}, \quad -1 \leq t \leq 1, \quad \text{and} \quad t = 4x^2 - 1.$

For $\frac{1}{2} < x^2 < 1$, it uses

$$\operatorname{arctanh} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right).$$

For $|x| \geq 1$, the routine fails as $\operatorname{arctanh} x$ is undefined.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – REAL (KIND=nag_wp) *Input*

On entry: the argument x of the function.

Constraint: $|X| < 1.0$.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6   Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

>   The routine has been called with an argument greater than or equal to 1.0 in magnitude, for which arctanh is not defined. On softfailure, the result is returned as zero.

IFAIL $= -99$

>   An unexpected error has been triggered by this routine. Please contact NAG.

>   See Section 3.8 in the Essential Introduction for further information.

IFAIL $= -399$

>   Your licence key may have expired or may not have been installed correctly.

>   See Section 3.7 in the Essential Introduction for further information.

IFAIL $= -999$

>   Dynamic memory allocation failed.

>   See Section 3.6 in the Essential Introduction for further information.

7   Accuracy

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{(1 - x^2)\,\mathrm{arctanh}\,x} \times \delta \right|.$$

That is, the relative error in the argument, x, is amplified by at least a factor $\dfrac{x}{(1 - x^2)\,\mathrm{arctanh}\,x}$ in the result. The equality should hold if δ is greater than the ***machine precision*** (δ due to data errors etc.) but if δ is simply due to round-off in the machine representation then it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the amplification factor is shown in the following graph:
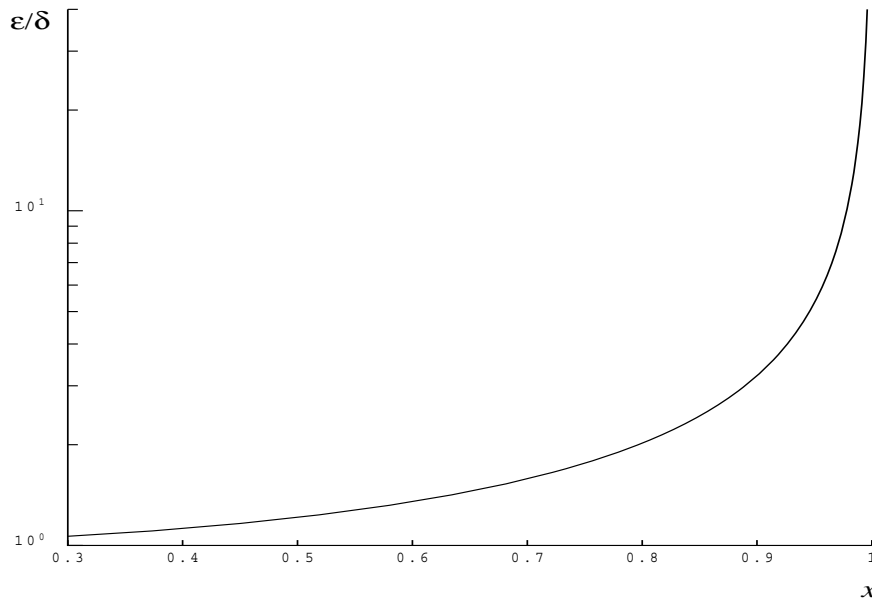
Figure 1

The factor is not significantly greater than one except for arguments close to $|x| = 1$. However in the region where $|x|$ is close to one, $1 - |x| \sim \delta$, the above analysis is inapplicable since x is bounded by definition, $|x| < 1$. In this region where arctanh is tending to infinity we have

$$\epsilon \sim 1/\ln \delta$$

which implies an obvious, unavoidable serious loss of accuracy near $|x| \sim 1$, e.g., if x and 1 agree to 6 significant figures, the result for arctanh x would be correct to at most about one figure.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
      Program s11aafe

!     S11AAF Example Program Text

!     Mark 25 Release. NAG Copyright 2014.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, s11aaf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: x, y
      Integer                          :: ifail, ioerr
!     .. Executable Statements ..
```

```
      Write (nout,*) 'S11AAF Example Program Results'

!     Skip heading in data file
      Read (nin,*)

      Write (nout,*)
      Write (nout,*) '     X           Y'
      Write (nout,*)

data: Do
        Read (nin,*,Iostat=ioerr) x

        If (ioerr<0) Then
          Exit data
        End If

        ifail = -1
        y = s11aaf(x,ifail)

        If (ifail<0) Then
          Exit data
        End If

        Write (nout,99999) x, y
      End Do data

99999 Format (1X,1P,2E12.3)
    End Program s11aafe
```

10.2 Program Data

```
S11AAF Example Program Data
          -0.5
           0.0
           0.5
          -0.9999
```

10.3 Program Results

```
 S11AAF Example Program Results

     X           Y

  -5.000E-01  -5.493E-01
   0.000E+00   0.000E+00
   5.000E-01   5.493E-01
  -9.999E-01  -4.952E+00
```