NAG Library Routine Document

G02BJF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

1 Purpose

G02BJF computes means and standard deviations, sums of squares and cross-products of deviations from means, and Pearson product-moment correlation coefficients for selected variables omitting cases with missing values from only those calculations involving the variables for which the values are missing.

2 Specification

3 Description

The input data consists of n observations for each of m variables, given as an array

$$[x_{ij}], \quad i = 1, 2, \dots, n (n \ge 2), j = 1, 2, \dots, m (m \ge 2),$$

where x_{ij} is the *i*th observation on the *j*th variable, together with the subset of these variables, v_1, v_2, \ldots, v_p , for which information is required.

In addition, each of the m variables may optionally have associated with it a value which is to be considered as representing a missing observation for that variable; the missing value for the jth variable is denoted by xm_j . Missing values need not be specified for all variables.

Let $w_{ij} = 0$ if the *i*th observation for the *j*th variable is a missing value, i.e., if a missing value, xm_j , has been declared for the *j*th variable, and $x_{ij} = xm_j$ (see also Section 7); and $w_{ij} = 1$ otherwise, for i = 1, 2, ..., n and j = 1, 2, ..., m.

The quantities calculated are:

(a) Means:

$$\bar{x}_j = \frac{\sum_{i=1}^n w_{ij} x_{ij}}{\sum_{i=1}^n w_{ij}}, \quad j = v_1, v_2, \dots, v_p.$$

(b) Standard deviations:

$$s_{j} = \sqrt{\frac{\sum_{i=1}^{n} w_{ij} (x_{ij} - \bar{x}_{j})^{2}}{\sum_{i=1}^{n} w_{ij} - 1}}, \quad j = v_{1}, v_{2}, \dots, v_{p}.$$

(c) Sums of squares and cross-products of deviations from means:

$$S_{jk} = \sum_{i=1}^{n} w_{ij} w_{ik} (x_{ij} - \bar{x}_{j(k)}) (x_{ik} - \bar{x}_{k(j)}), \quad j, k = v_1, v_2, \dots, v_p,$$

where

$$ar{x}_{j(k)} = rac{\displaystyle\sum_{i=1}^{n} w_{ij} w_{ik} x_{ij}}{\displaystyle\sum_{i=1}^{n} w_{ij} w_{ik}} \quad ext{ and } \quad ar{x}_{k(j)} = rac{\displaystyle\sum_{i=1}^{n} w_{ik} w_{ij} x_{ik}}{\displaystyle\sum_{i=1}^{n} w_{ik} w_{ij}},$$

(i.e., the means used in the calculation of the sum of squares and cross-products of deviations are based on the same set of observations as are the cross-products).

(d) Pearson product-moment correlation coefficients:

$$R_{jk} = \frac{S_{jk}}{\sqrt{S_{jj(k)}S_{kk(j)}}}, \quad j, k = v_1, v_2, \dots, v_p,$$

where

$$S_{jj(k)} = \sum_{i=1}^{n} w_{ij} w_{ik} (x_{ij} - \bar{x}_{j(k)})^2$$
 and $S_{kk(j)} = \sum_{i=1}^{n} w_{ik} w_{ij} (x_{ik} - \bar{x}_{k(j)})^2$,

(i.e., the sums of squares of deviations used in the denominator are based on the same set of observations as are used in the calculation of the numerator).

If $S_{jj(k)}$ or $S_{kk(j)}$ is zero, R_{jk} is set to zero.

(e) The number of cases used in the calculation of each of the correlation coefficients:

$$c_{jk} = \sum_{i=1}^{n} w_{ij} w_{ik}, \quad j, k = v_1, v_2, \dots, v_p.$$

(The diagonal terms, c_{jj} , for $j=v_1,v_2,\ldots,v_p$, also give the number of cases used in the calculation of the means, \bar{x}_j , and the standard deviations, s_j .)

4 References

None.

5 Parameters

1: N – INTEGER Input

On entry: n, the number of observations or cases.

Constraint: $N \ge 2$.

2: M – INTEGER Input

On entry: m, the number of variables.

Constraint: $M \ge 2$.

3: $X(LDX, M) - REAL (KIND=nag_wp)$ array Input

On entry: X(i,j) must be set to x_{ij} , the value of the *i*th observation on the *j*th variable, for i = 1, 2, ..., n and j = 1, 2, ..., m.

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4: LDX – INTEGER

Input

On entry: the first dimension of the array X as declared in the (sub)program from which G02BJF is called.

Constraint: LDX \geq N.

5: MISS(M) – INTEGER array

Input

On entry: MISS(j) must be set equal to 1 if a missing value, xm_j , is to be specified for the jth variable in the array X, or set equal to 0 otherwise. Values of MISS must be given for all m variables in the array X.

6: XMISS(M) – REAL (KIND=nag wp) array

Input

On entry: XMISS(j) must be set to the missing value, xm_j , to be associated with the jth variable in the array X, for those variables for which missing values are specified by means of the array MISS (see Section 7).

7: NVARS – INTEGER

Input

On entry: p, the number of variables for which information is required.

Constraint: $2 \le NVARS \le M$.

8: KVAR(NVARS) – INTEGER array

Input

On entry: KVAR(j) must be set to the column number in X of the jth variable for which information is required, for j = 1, 2, ..., p.

Constraint: $1 \leq \text{KVAR}(j) \leq M$, for j = 1, 2, ..., p.

9: XBAR(NVARS) - REAL (KIND=nag_wp) array

Output

On exit: the mean value, \bar{x}_j , of the variable specified in KVAR(j), for $j = 1, 2, \dots, p$.

10: STD(NVARS) – REAL (KIND=nag wp) array

Output

On exit: the standard deviation, s_j , of the variable specified in KVAR(j), for j = 1, 2, ..., p.

11: SSP(LDSSP, NVARS) - REAL (KIND=nag wp) array

Output

On exit: SSP(j, k) is the cross-product of deviations, S_{jk} , for the variables specified in KVAR(j) and KVAR(k), for j = 1, 2, ..., p and k = 1, 2, ..., p.

12: LDSSP – INTEGER

Input

On entry: the first dimension of the array SSP as declared in the (sub)program from which G02BJF is called.

Constraint: LDSSP \geq NVARS.

13: R(LDR, NVARS) - REAL (KIND=nag wp) array

Output

On exit: R(j, k) is the product-moment correlation coefficient, R_{jk} , between the variables specified in KVAR(j) and KVAR(k), for j = 1, 2, ..., p and k = 1, 2, ..., p.

14: LDR – INTEGER

Input

On entry: the first dimension of the array R as declared in the (sub)program from which G02BJF is called.

Constraint: LDR \geq NVARS.

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15: NCASES - INTEGER

Output

On exit: the minimum number of cases used in the calculation of any of the sums of squares and cross-products and correlation coefficients (when cases involving missing values have been eliminated).

16: CNT(LDCNT, NVARS) - REAL (KIND=nag_wp) array

Output

On exit: CNT(j, k) is the number of cases, c_{jk} , actually used in the calculation of S_{jk} , and R_{jk} , the sum of cross-products and correlation coefficient for the variables specified in KVAR(j) and KVAR(k), for j = 1, 2, ..., p and k = 1, 2, ..., p.

17: LDCNT - INTEGER

Input

On entry: the first dimension of the array CNT as declared in the (sub)program from which G02BJF is called.

Constraint: LDCNT \geq NVARS.

18: IFAIL - INTEGER

Input/Output

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output parameters may be useful even if IFAIL $\neq 0$ on exit, the recommended value is -1. When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.

On exit: IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Note: G02BJF may return useful information for one or more of the following detected errors or warnings.

Errors or warnings detected by the routine:

```
IFAIL = 1
     On entry, N < 2.
IFAIL = 2
     On entry, NVARS < 2,
              NVARS > M.
IFAIL = 3
     On entry, LDX < N,
              LDSSP < NVARS,
     or
              LDR < NVARS,
     or
              LDCNT < NVARS.
     or
IFAIL = 4
     On entry, KVAR(j) < 1,
              KVAR(j) > M for some j = 1, 2, ..., NVARS.
```

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IFAIL = 5

After observations with missing values were omitted, fewer than two cases remained for at least one pair of variables. (The pairs of variables involved can be determined by examination of the contents of the array CNT.) All means, standard deviations, sums of squares and cross-products, and correlation coefficients based on two or more cases are returned by the routine even if IFAIL = 5.

IFAIL = -99

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.8 in the Essential Introduction for further information.

IFAIL = -399

Your licence key may have expired or may not have been installed correctly.

See Section 3.7 in the Essential Introduction for further information.

IFAIL = -999

Dynamic memory allocation failed.

See Section 3.6 in the Essential Introduction for further information.

7 Accuracy

G02BJF does not use *additional precision* arithmetic for the accumulation of scalar products, so there may be a loss of significant figures for large n.

You are warned of the need to exercise extreme care in your selection of missing values. G02BJF treats all values in the inclusive range $(1 \pm 0.1^{(\text{X02BEF}-2)}) \times xm_j$, where xm_j is the missing value for variable j specified in XMISS.

You must therefore ensure that the missing value chosen for each variable is sufficiently different from all values for that variable so that none of the valid values fall within the range indicated above.

8 Parallelism and Performance

Not applicable.

9 Further Comments

The time taken by G02BJF depends on n and p, and the occurrence of missing values.

The routine uses a two-pass algorithm.

10 Example

This example reads in a set of data consisting of five observations on each of four variables. Missing values of -1.0, 0.0 and 0.0 are declared for the first, second and fourth variables respectively; no missing value is specified for the third variable. The means, standard deviations, sums of squares and cross-products of deviations from means, and Pearson product-moment correlation coefficients for the fourth, first and second variables are then calculated and printed, omitting cases with missing values from only those calculations involving the variables for which the values are missing. The program therefore eliminates cases 4 and 5 in calculating the correlation between the fourth and first variables, and cases 3 and 4 for the fourth and second variables etc.

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10.1 Program Text

```
Program g02bjfe
     GO2BJF Example Program Text
!
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1
      .. Use Statements ..
!
     Use nag_library, Only: g02bjf, nag_wp
!
      .. Implicit None Statement ..
     Implicit None
!
      .. Parameters ..
                                       :: nin = 5, nout = 6
     Integer, Parameter
      .. Local Scalars ..
                                       :: i, ifail, ldcnt, ldr, ldssp, ldx, m, &
     Integer
                                          n, ncases, nvars
1
      .. Local Arrays ..
     Real (Kind=nag_wp), Allocatable :: cnt(:,:), r(:,:), ssp(:,:), std(:), &
                                          x(:,:), xbar(:), xmiss(:)
     Integer, Allocatable
                                       :: kvar(:), miss(:)
!
      .. Executable Statements ..
     Write (nout,*) 'G02BJF Example Program Results'
     Write (nout,*)
     Skip heading in data file
     Read (nin,*)
     Read in the problem size
1
     Read (nin,*)^-n, m, nvars
     ldcnt = nvars
     ldr = nvars
     ldssp = nvars
     ldx = n
     Allocate (cnt(ldcnt,nvars),r(ldr,nvars),ssp(ldssp,nvars),std(nvars), &
       x(ldx,m),xbar(nvars),xmiss(m),kvar(nvars),miss(m))
     Read in data
     Read (nin,*)(x(i,1:m),i=1,n)
1
     Read in missing value flags
     Read (nin,*) miss(1:m)
     Read (nin,*) xmiss(1:m)
     Read in column IDs
     Read (nin,*) kvar(1:nvars)
     Display data
     Write (nout,99999) 'Number of variables (columns) =', m
     Write (nout,99999) 'Number of cases (rows) =', n
     Write (nout,*)
Write (nout,*) 'Data matrix is:-'
     Write (nout, 99998)(i, i=1, m)
     Write (nout, 99997) (i, x(i, 1:m), i=1, n)
     Write (nout,*)
     Calculate summary statistics
     ifail = 0
     Call g02bjf(n,m,x,ldx,miss,xmiss,nvars,kvar,xbar,std,ssp,ldssp,r,ldr, &
       ncases,cnt,ldcnt,ifail)
     Display results
     Write (nout,*) 'Variable Mean St. dev.'
     Write (nout,99995)(kvar(i),xbar(i),std(i),i=1,nvars)
     Write (nout,*)
     Write (nout,*) 'Sums of squares and cross-products of deviations'
     Write (nout, 99998) kvar(1:nvars)
     Write (nout,99996)(kvar(i),ssp(i,1:nvars),i=1,nvars)
     Write (nout,*)
Write (nout,*) 'Correlation coefficients'
```

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```
Write (nout,99998) kvar(1:nvars)
Write (nout,99996)(kvar(i),r(i,1:nvars),i=1,nvars)
Write (nout,*)
Write (nout,99999) &
    'Minimum number of cases used for any pair of variables:', ncases
Write (nout,*)
Write (nout,*) 'Numbers used for each pair are:'
Write (nout,99998) kvar(1:nvars)
Write (nout,99996)(kvar(i),cnt(i,1:nvars),i=1,nvars)

99999 Format (1X,A,I5)
99998 Format (1X,4I12)
99997 Format (1X,I3,4F12.4)
99996 Format (1X,I3,3F12.4)
99995 Format (1X,I5,2F11.4)
End Program g02bjfe
```

10.2 Program Data

```
GO2BJF Example Program Data
5 4 3
                       :: N, M, NVARS
                 2.0
3.0
           1.0
      3.0
                  4.0
          -1.0
6.0
      4.0
     0.0
           5.0
9.0
                  9.0
12.0
    2.0
5.0
1 0
      2.0
            0.0
                  0.0
-1.0
                  12.0 :: End of X
1
                  1 :: MISS
-1.0 0.0 0.0
4 1 2
                  0.0 :: XMISS
                       :: KVAR
```

10.3 Program Results

```
GO2BJF Example Program Results
```

```
Number of variables (columns) = 4
Number of cases (rows) = 5
```

```
Data matrix is:-
                2
3.0000
4.000
                               3
                          1.0000
                                       2.0000
        3.0000
 1
                  4.0000
0.0000
 2
        6.0000
                             -1.0000
                                         4.0000
                                        9.0000
                            5.0000
       9.0000
 3
                2.0000
                                        0.0000
 4
       12.0000
                             4.0000 12.0000
 5
       -1.0000
                  5.0000
```

```
Variable Mean St. dev.
4 6.7500 4.5735
1 7.5000 3.8730
2 3.5000 1.2910
```

Sums of squares and cross-products of deviations

```
4 1 2
4 62.7500 21.0000 10.0000
1 21.0000 45.0000 -6.0000
2 10.0000 -6.0000 5.0000
```

Correlation coefficients

```
4 1 2
4 1.0000 0.9707 0.9449
1 0.9707 1.0000 -0.6547
2 0.9449 -0.6547 1.0000
```

Minimum number of cases used for any pair of variables:

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Numbers	used for	each pair are:	
	4	1	2
4	4.0000	3.0000	3.0000
1	3.0000	4.0000	3.0000
2	3.0000	3.0000	4.0000

G02BJF.8 (last)

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