NAG Library Routine Document

G01EEF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G01EEF computes the upper and lower tail probabilities and the probability density function of the beta distribution with parameters a and b.

2 Specification

```
SUBROUTINE G01EEF (X, A, B, TOL, P, Q, PDF, IFAIL)
INTEGER            IFAIL
REAL (KIND=nag_wp) X, A, B, TOL, P, Q, PDF
```

3 Description

The probability density function of the beta distribution with parameters a and b is:

$$f(B : a, b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} B^{a-1}(1-B)^{b-1}, \quad 0 \leq B \leq 1; a, b > 0.$$

The lower tail probability, $P(B \leq \beta : a, b)$ is defined by

$$P(B \leq \beta : a, b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^\beta B^{a-1}(1-B)^{b-1}\, dB = I_\beta(a, b), \quad 0 \leq \beta \leq 1; a, b > 0.$$

The function $I_x(a, b)$, also known as the incomplete beta function is calculated using S14CCF.

4 References

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

5 Parameters

1: X – REAL (KIND=nag_wp) *Input*

On entry: β, the value of the beta variate.

Constraint: $0.0 \leq \mathrm{X} \leq 1.0$.

2: A – REAL (KIND=nag_wp) *Input*

On entry: a, the first parameter of the required beta distribution.

Constraint: $0.0 < \mathrm{A} \leq 10^6$.

3: B – REAL (KIND=nag_wp) *Input*

On entry: b, the second parameter of the required beta distribution.

Constraint: $0.0 < \mathrm{B} \leq 10^6$.

4: TOL – REAL (KIND=nag_wp) *Input*

On entry: this parameter is no longer referenced, but is included for backwards compatability.

5: P – REAL (KIND=nag_wp) *Output*

On exit: the lower tail probability, $P(B \leq \beta : a, b)$.

6: Q – REAL (KIND=nag_wp) *Output*

On exit: the upper tail probability, $P(B \geq \beta : a, b)$.

7: PDF – REAL (KIND=nag_wp) *Output*

On exit: the probability density function, $f(B : a, b)$.

8: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output parameters may be useful even if IFAIL $\neq 0$ on exit, the recommended value is -1. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Note: G01EEF may return useful information for one or more of the following detected errors or warnings.

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, X < 0.0,
or X > 1.0.

IFAIL $= 2$

On entry, A ≤ 0.0,
or A $> 10^6$,
or B ≤ 0.0,
or B $> 10^6$.

IFAIL $= 4$

X is too far out into the tails for the probability to be evaluated exactly. The results returned are 0 and 1 as appropriate. These should be a good approximation to the required solution.

IFAIL $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.8 in the Essential Introduction for further information.

IFAIL $= -399$

Your licence key may have expired or may not have been installed correctly.

See Section 3.7 in the Essential Introduction for further information.

IFAIL $= -999$

Dynamic memory allocation failed.

See Section 3.6 in the Essential Introduction for further information.

7 Accuracy

The accuracy is limited by the error in the incomplete beta function. See Section 7 in S14CCF for further details.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values from a number of beta distributions and computes the associated upper and lower tail probabilities and the corresponding value of the probability density function.

10.1 Program Text

```
    Program g01eefe

!     G01EEF Example Program Text

!     Mark 25 Release. NAG Copyright 2014.

!     .. Use Statements ..
      Use nag_library, Only: g01eef, nag_wp
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: a, b, p, pdf, q, tol, x
      Integer                          :: ifail
!     .. Executable Statements ..
      Write (nout,*) 'G01EEF Example Program Results'
      Write (nout,*)

!     Skip heading in data file
      Read (nin,*)

!     Display titles
      Write (nout,*) '    X        A        B        P        Q       PDF'
      Write (nout,*)

d_lp: Do
        Read (nin,*,Iostat=ifail) x, a, b
        If (ifail/=0) Then
          Exit d_lp
        End If

!       Calculate probability
!       NB: parameter tol is no longer referenced
        ifail = -1
        Call g01eef(x,a,b,tol,p,q,pdf,ifail)
        If (ifail/=0) Then
          If (ifail/=3 .And. ifail/=4) Then
            Exit d_lp
```

```
          End If
        End If

!       Display results
        Write (nout,99999) x, a, b, p, q, pdf
      End Do d_lp

99999 Format (1X,6(F7.4,2X),A,I1)
    End Program g01eefe
```

10.2 Program Data

```
G01EEF Example Program Data
 0.25  1.0  2.0
 0.75  1.5  1.5
 0.5   2.0  1.0
```

10.3 Program Results

```
 G01EEF Example Program Results

     X        A        B        P        Q       PDF

  0.2500   1.0000   2.0000   0.4375   0.5625   1.5000
  0.7500   1.5000   1.5000   0.8045   0.1955   1.1027
  0.5000   2.0000   1.0000   0.2500   0.7500   1.0000
```