# NAG Library Routine Document F08WJF (DGGBAK) 

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms
and other implementation-dependent details.

## 1 Purpose

F08WJF (DGGBAK) forms the right or left eigenvectors of the real generalized eigenvalue problem $A x=\lambda B x$, by backward transformation on the computed eigenvectors given by F08YKF (DTGEVC). It is necessary to call this routine only if the optional balancing routine F08WHF (DGGBAL) was previously called to balance the matrix pair $(A, B)$.

## 2 Specification

```
SUBROUTINE FO8WJF (JOB, SIDE, N, ILO, IHI, LSCALE, RSCALE, M, V, LDV,
                        INFO)
INTEGER N, ILO, IHI, M, LDV, INFO
REAL (KIND=nag_wp) LSCALE (*), RSCALE(*), V(LDV,*)
CHARACTER(1) JOB, SIDE
```

The routine may be called by its LAPACK name dggbak.

## 3 Description

If the matrix pair has been previously balanced using the routine F08WHF (DGGBAL) then F08WJF (DGGBAK) backtransforms the eigenvector solution given by F08YKF (DTGEVC). This is usually the sixth and last step in the solution of the generalized eigenvalue problem.

For a description of balancing, see the document for F08WHF (DGGBAL).

## 4 References

Ward R C (1981) Balancing the generalized eigenvalue problem SIAM J. Sci. Stat. Comp. 2 141-152

## 5 Parameters

1: JOB - CHARACTER(1)
On entry: specifies the backward transformation step required.
$\mathrm{JOB}=\mathrm{N}^{\prime}{ }^{\prime}$
No transformations are done.
$\mathrm{JOB}=\mathrm{P}^{\prime}$
Only do backward transformations based on permutations.
$\mathrm{JOB}=\mathrm{S}^{\prime}$
Only do backward transformations based on scaling.
$\mathrm{JOB}=$ 'B'
Do backward transformations for both permutations and scaling.
Note: this must be the same parameter JOB as supplied to F08WHF (DGGBAL).
Constraint: $\mathrm{JOB}=\mathrm{N}^{\prime}$ ', 'P', 'S' or 'B'.

2: SIDE - CHARACTER(1)
Input
On entry: indicates whether left or right eigenvectors are to be transformed.
SIDE $=$ 'L'
The left eigenvectors are transformed.
SIDE $=$ ' $\mathrm{R}^{\prime}$
The right eigenvectors are transformed.
Constraint: SIDE $=$ 'L' or 'R'.
3: N - INTEGER
Input
On entry: $n$, the order of the matrices $A$ and $B$ of the generalized eigenvalue problem.
Constraint: $\mathrm{N} \geq 0$.
4: ILO - INTEGER Input
5: IHI - INTEGER Input
On entry: $i_{\text {lo }}$ and $i_{\text {hi }}$ as determined by a previous call to F08WHF (DGGBAL).
Constraints:

$$
\begin{aligned}
& \text { if } \mathrm{N}>0,1 \leq \mathrm{ILO} \leq \mathrm{IHI} \leq \mathrm{N} \\
& \text { if } \mathrm{N}=0, \mathrm{ILO}=1 \text { and } \mathrm{IHI}=0
\end{aligned}
$$

6: $\operatorname{LSCALE}(*)$ - REAL (KIND=nag_wp) array Input
Note: the dimension of the array LSCALE must be at least $\max (1, \mathrm{~N})$.
On entry: details of the permutations and scaling factors applied to the left side of the matrices $A$ and $B$, as returned by a previous call to F08WHF (DGGBAL).

7: $\operatorname{RSCALE}(*)$ - REAL (KIND=nag_wp) array Input
Note: the dimension of the array RSCALE must be at least $\max (1, N)$.
On entry: details of the permutations and scaling factors applied to the right side of the matrices $A$ and $B$, as returned by a previous call to F08WHF (DGGBAL).

8: M - INTEGER
Input
On entry: $m$, the required number of left or right eigenvectors.
Constraint: $0 \leq \mathrm{M} \leq \mathrm{N}$.
9: $\quad \mathrm{V}(\mathrm{LDV}, *)$ - REAL (KIND=nag_wp) array
Input/Output
Note: the second dimension of the array V must be at least $\max (1, \mathrm{M})$.
On entry: the matrix of right or left eigenvectors, as returned by F08WHF (DGGBAL).
On exit: the transformed right or left eigenvectors.
10: LDV - INTEGER
Input
On entry: the first dimension of the array V as declared in the (sub)program from which F08WJF (DGGBAK) is called.
Constraint: $\operatorname{LDV} \geq \max (1, \mathrm{~N})$.
11: INFO - INTEGER
Output
On exit: $\mathrm{INFO}=0$ unless the routine detects an error (see Section 6).

## 6 Error Indicators and Warnings

INFO $<0$
If INFO $=-i$, argument $i$ had an illegal value. An explanatory message is output, and execution of the program is terminated.

## 7 Accuracy

The errors are negligible, compared with the previous computations.

## 8 Parallelism and Performance

F08WJF (DGGBAK) is not threaded by NAG in any implementation.
F08WJF (DGGBAK) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.
Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The number of operations is proportional to $n^{2}$.
The complex analogue of this routine is F08WWF (ZGGBAK).

## 10 Example

See Section 10 in F08XEF (DHGEQZ) and F08YKF (DTGEVC).

