NAG Library Routine Document

F08WEF (DGGHRD)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F08WEF (DGGHRD) reduces a pair of real matrices (A, B), where B is upper triangular, to the generalized upper Hessenberg form using orthogonal transformations.

2 Specification

```
SUBROUTINE F08WEF (COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB, Q, LDQ, Z,    &
                   LDZ, INFO)
INTEGER            N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO
REAL (KIND=nag_wp) A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*)
CHARACTER(1)       COMPQ, COMPZ
```

The routine may be called by its LAPACK name ***dgghrd***.

3 Description

F08WEF (DGGHRD) is the third step in the solution of the real generalized eigenvalue problem

$$Ax = \lambda Bx.$$

The (optional) first step balances the two matrices using F08WHF (DGGBAL). In the second step, matrix B is reduced to upper triangular form using the QR factorization routine F08AEF (DGEQRF) and this orthogonal transformation Q is applied to matrix A by calling F08AGF (DORMQR).

F08WEF (DGGHRD) reduces a pair of real matrices (A, B), where B is upper triangular, to the generalized upper Hessenberg form using orthogonal transformations. This two-sided transformation is of the form

$$Q^{\mathrm{T}}AZ = H$$
$$Q^{\mathrm{T}}BZ = T$$

where H is an upper Hessenberg matrix, T is an upper triangular matrix and Q and Z are orthogonal matrices determined as products of Givens rotations. They may either be formed explicitly, or they may be postmultiplied into input matrices Q_1 and Z_1, so that

$$Q_1AZ_1^{\mathrm{T}} = (Q_1Q)H(Z_1Z)^{\mathrm{T}},$$
$$Q_1BZ_1^{\mathrm{T}} = (Q_1Q)T(Z_1Z)^{\mathrm{T}}.$$

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Moler C B and Stewart G W (1973) An algorithm for generalized matrix eigenproblems *SIAM J. Numer. Anal.* **10** 241–256

5 Parameters

1: COMPQ – CHARACTER(1) *Input*

On entry: specifies the form of the computed orthogonal matrix Q.

COMPQ = 'N'
Do not compute Q.

COMPQ = 'I'
The orthogonal matrix Q is returned.

COMPQ = 'V'
Q must contain an orthogonal matrix Q_1, and the product $Q_1 Q$ is returned.

Constraint: COMPQ = 'N', 'I' or 'V'.

2: COMPZ – CHARACTER(1) *Input*

On entry: specifies the form of the computed orthogonal matrix Z.

COMPZ = 'N'
Do not compute Z.

COMPZ = 'I'
The orthogonal matrix Z is returned.

COMPZ = 'V'
Z must contain an orthogonal matrix Z_1, and the product $Z_1 Z$ is returned.

Constraint: COMPZ = 'N', 'V' or 'I'.

3: N – INTEGER *Input*

On entry: n, the order of the matrices A and B.

Constraint: $\mathrm{N} \geq 0$.

4: ILO – INTEGER *Input*
5: IHI – INTEGER *Input*

On entry: i_{lo} and i_{hi} as determined by a previous call to F08WHF (DGGBAL). Otherwise, they should be set to 1 and n, respectively.

Constraints:

if $\mathrm{N} > 0$, $1 \leq \mathrm{ILO} \leq \mathrm{IHI} \leq \mathrm{N}$;
if $\mathrm{N} = 0$, $\mathrm{ILO} = 1$ and $\mathrm{IHI} = 0$.

6: A(LDA, $*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array A must be at least $\max(1, \mathrm{N})$.

On entry: the matrix A of the matrix pair (A, B). Usually, this is the matrix A returned by F08AGF (DORMQR).

On exit: A is overwritten by the upper Hessenberg matrix H.

7: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F08WEF (DGGHRD) is called.

Constraint: $\mathrm{LDA} \geq \max(1, \mathrm{N})$.

8: B(LDB, $*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array B must be at least max(1, N).

On entry: the upper triangular matrix B of the matrix pair (A, B). Usually, this is the matrix B returned by the QR factorization routine F08AEF (DGEQRF).

On exit: B is overwritten by the upper triangular matrix T.

9: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F08WEF (DGGHRD) is called.

Constraint: LDB $\geq$ max(1, N).

10: Q(LDQ, $*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array Q must be at least max(1, N) if COMPQ = 'I' or 'V' and at least 1 if COMPQ = 'N'.

On entry: if COMPQ = 'V', Q must contain an orthogonal matrix Q_1.

If COMPQ = 'N', Q is not referenced.

On exit: if COMPQ = 'I', Q contains the orthogonal matrix Q.

If COMPQ = 'V', Q is overwritten by Q_1Q.

11: LDQ – INTEGER *Input*

On entry: the first dimension of the array Q as declared in the (sub)program from which F08WEF (DGGHRD) is called.

Constraints:

if COMPQ = 'I' or 'V', LDQ $\geq$ max(1, N);
if COMPQ = 'N', LDQ $\geq$ 1.

12: Z(LDZ, $*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array Z must be at least max(1, N) if COMPZ = 'V' or 'I' and at least 1 if COMPZ = 'N'.

On entry: if COMPZ = 'V', Z must contain an orthogonal matrix Z_1.

If COMPZ = 'N', Z is not referenced.

On exit: if COMPZ = 'I', Z contains the orthogonal matrix Z.

If COMPZ = 'V', Z is overwritten by Z_1Z.

13: LDZ – INTEGER *Input*

On entry: the first dimension of the array Z as declared in the (sub)program from which F08WEF (DGGHRD) is called.

Constraints:

if COMPZ = 'V' or 'I', LDZ $\geq$ max(1, N);
if COMPZ = 'N', LDZ $\geq$ 1.

14: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The reduction to the generalized Hessenberg form is implemented using orthogonal transformations which are backward stable.

8 Parallelism and Performance

F08WEF (DGGHRD) is not threaded by NAG in any implementation.

F08WEF (DGGHRD) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

This routine is usually followed by F08XEF (DHGEQZ) which implements the QZ algorithm for computing generalized eigenvalues of a reduced pair of matrices.

The complex analogue of this routine is F08WSF (ZGGHRD).

10 Example

See Section 10 in F08XEF (DHGEQZ) and F08YKF (DTGEVC).