

NAG Library Routine Document

F07PJF (DSPTRI)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F07PJF (DSPTRI) computes the inverse of a real symmetric indefinite matrix A , where A has been factorized by F07PDF (DSPTRF), using packed storage.

2 Specification

```
SUBROUTINE F07PJF (UPLO, N, AP, IPIV, WORK, INFO)
INTEGER          N, IPIV(*), INFO
REAL (KIND=nag_wp) AP(*), WORK(N)
CHARACTER(1)    UPLO
```

The routine may be called by its LAPACK name *dsptri*.

3 Description

F07PJF (DSPTRI) is used to compute the inverse of a real symmetric indefinite matrix A , the routine must be preceded by a call to F07PDF (DSPTRF), which computes the Bunch–Kaufman factorization of A , using packed storage.

If UPLO = 'U', $A = PUDU^T P^T$ and A^{-1} is computed by solving $U^T P^T X P U = D^{-1}$.

If UPLO = 'L', $A = PLDL^T P^T$ and A^{-1} is computed by solving $L^T P^T X P L = D^{-1}$.

4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

- 1: UPLO – CHARACTER(1) *Input*
On entry: specifies how A has been factorized.
 UPLO = 'U'
 $A = PUDU^T P^T$, where U is upper triangular.
 UPLO = 'L'
 $A = PLDL^T P^T$, where L is lower triangular.
Constraint: UPLO = 'U' or 'L'.
- 2: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.
- 3: AP(*) – REAL (KIND=nag_wp) array *Input/Output*
Note: the dimension of the array AP must be at least $\max(1, N \times (N + 1)/2)$.
On entry: the factorization of A stored in packed form, as returned by F07PDF (DSPTRF).

On exit: the factorization is overwritten by the n by n matrix A^{-1} .

More precisely,

if UPLO = 'U', the upper triangle of A^{-1} must be stored with element A_{ij} in $AP(i + j(j - 1)/2)$ for $i \leq j$;

if UPLO = 'L', the lower triangle of A^{-1} must be stored with element A_{ij} in $AP(i + (2n - j)(j - 1)/2)$ for $i \geq j$.

4: IPIV(*) – INTEGER array *Input*

Note: the dimension of the array IPIV must be at least $\max(1, N)$.

On entry: details of the interchanges and the block structure of D , as returned by F07PDF (DSPTRF).

5: WORK(N) – REAL (KIND=nag_wp) array *Workspace*

6: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO = $-i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

Element $\langle value \rangle$ of the diagonal is exactly zero. D is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies a bound of the form

if UPLO = 'U', $|DU^T P^T X P U - I| \leq c(n)\epsilon(|D||U^T|P^T|X|P|U| + |D||D^{-1}|)$;

if UPLO = 'L', $|DL^T P^T X P L - I| \leq c(n)\epsilon(|D||L^T|P^T|X|P|L| + |D||D^{-1}|)$,

$c(n)$ is a modest linear function of n , and ϵ is the *machine precision*.

8 Parallelism and Performance

F07PJF (DSPTRI) is not threaded by NAG in any implementation.

F07PJF (DSPTRI) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}n^3$.

The complex analogues of this routine are F07PWF (ZHPTRI) for Hermitian matrices and F07QWF (ZSPTRI) for symmetric matrices.

10 Example

This example computes the inverse of the matrix A , where

$$A = \begin{pmatrix} 2.07 & 3.87 & 4.20 & -1.15 \\ 3.87 & -0.21 & 1.87 & 0.63 \\ 4.20 & 1.87 & 1.15 & 2.06 \\ -1.15 & 0.63 & 2.06 & -1.81 \end{pmatrix}.$$

Here A is symmetric indefinite, stored in packed form, and must first be factorized by F07PDF (DSPTRF).

10.1 Program Text

```

Program f07pjfe

!       F07PJF Example Program Text

!       Mark 25 Release. NAG Copyright 2014.

!       .. Use Statements ..
Use nag_library, Only: dsptrf, dsptri, nag_wp, x04ccf
!       .. Implicit None Statement ..
Implicit None
!       .. Parameters ..
Integer, Parameter          :: nin = 5, nout = 6
!       .. Local Scalars ..
Integer                    :: i, ifail, info, j, n
Character (1)              :: uplo
!       .. Local Arrays ..
Real (Kind=nag_wp), Allocatable :: ap(:), work(:)
Integer, Allocatable       :: ipiv(:)
!       .. Executable Statements ..
Write (nout,*) 'F07PJF Example Program Results'
!       Skip heading in data file
Read (nin,*)
Read (nin,*) n

Allocate (ap(n*(n+1)/2),work(n),ipiv(n))

!       Read A from data file

Read (nin,*) uplo
If (uplo=='U') Then
  Read (nin,*)((ap(i+j*(j-1)/2),j=i,n),i=1,n)
Else If (uplo=='L') Then
  Read (nin,*)((ap(i+(2*n-j)*(j-1)/2),j=1,i),i=1,n)
End If

!       Factorize A
!       The NAG name equivalent of dsptrf is f07pdf
Call dsptrf(uplo,n,ap,ipiv,info)

Write (nout,*)
Flush (nout)
If (info==0) Then

!       Compute inverse of A
!       The NAG name equivalent of dsptri is f07pjf
Call dsptri(uplo,n,ap,ipiv,work,info)

!       Print inverse

!       ifail: behaviour on error exit
!             =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
ifail = 0
Call x04ccf(uplo,'Nonunit',n,ap,'Inverse',ifail)

```

```

Else
  Write (nout,*) 'The factor D is singular'
End If

End Program f07pjfe

```

10.2 Program Data

```

F07PJF Example Program Data
  4                               :Value of N
  'L'                             :Value of UPLO
  2.07
  3.87 -0.21
  4.20  1.87  1.15
  -1.15  0.63  2.06 -1.81 :End of matrix A

```

10.3 Program Results

F07PJF Example Program Results

```

Inverse
      1          2          3          4
1      0.7485
2      0.5221   -0.1605
3     -1.0058   -0.3131   1.3501
4     -1.4386   -0.7440   2.0667   2.4547

```
