

## NAG Library Chapter Contents

### s – Approximations of Special Functions

s Chapter Introduction

| Function Name | Mark of Introduction | Purpose   |
|---------------|----------------------|---|
| s01bac        | 7                    | nag_shifted_log<br>$\ln(1+x)$   |
| s10aac        | 1                    | nag_tanh<br>Hyperbolic tangent, $\tanh x$   |
| s10abc        | 1                    | nag_sinh<br>Hyperbolic sine, $\sinh x$  |
| s10acc        | 1                    | nag_cosh<br>Hyperbolic cosine, $\cosh x$  |
| s11aac        | 1                    | nag_arctanh<br>Inverse hyperbolic tangent, $\operatorname{arctanh} x$                             |
| s11abc        | 1                    | nag_arcsinh<br>Inverse hyperbolic sine, $\operatorname{arcsinh} x$                                |
| s11acc        | 1                    | nag_arccosh<br>Inverse hyperbolic cosine, $\operatorname{arccosh} x$                              |
| s13aac        | 1                    | nag_exp_integral<br>Exponential integral $E_1(x)$   |
| s13acc        | 1                    | nag_cos_integral<br>Cosine integral $\operatorname{Ci}(x)$  |
| s13adc        | 1                    | nag_sin_integral<br>Sine integral $\operatorname{Si}(x)$  |
| s14aac        | 1                    | nag_gamma<br>Gamma function $\Gamma(x)$   |
| s14abc        | 1                    | nag_log_gamma<br>Log gamma function $\ln(\Gamma(x))$  |
| s14acc        | 7                    | nag_polygamma_fun<br>$*(x) - \ln x$ where $*(x)$ is the psi function                              |
| s14adc        | 7                    | nag_polygamma_deriv<br>Scaled derivatives of $\psi(x)$  |
| s14aec        | 6                    | nag_real_polygamma<br>Derivative of the psi function $\psi(x)$                                    |
| s14afc        | 6                    | nag_complex_polygamma<br>Derivative of the psi function $\psi(z)$                                 |
| s14agc        | 7                    | nag_complex_log_gamma<br>Logarithm of the gamma function $\ln \Gamma(z)$ , complex argument       |
| s14ahc        | 9                    | nag_scaled_log_gamma<br>Scaled log gamma function $\ln G(x)$ , where $G(x) = \Gamma(x+1)/(x/e)^x$ |
| s14bac        | 1                    | nag_incomplete_gamma<br>Incomplete gamma functions $P(a, x)$ and $Q(a, x)$                        |
| s14cbc        | 23                   | nag_log_beta<br>Logarithm of the beta function $\ln B(a, b)$                                      |
| s14ccc        | 23                   | nag_incomplete_beta<br>Incomplete beta function $I_x(a, b)$ and its complement $1 - I_x$          |
| s15abc        | 1                    | nag_cumul_normal<br>Cumulative Normal distribution function $P(x)$                                |
| s15acc        | 1                    | nag_cumul_normal_complem<br>Complement of cumulative Normal distribution function $Q(x)$          |
| s15adc        | 1                    | nag_erfc<br>Complement of error function $\operatorname{erfc}(x)$                                 |

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|--------|----|---|
| s15aec | 1  | nag_erf<br>Error function $\operatorname{erf}(x)$   |
| s15afc | 7  | nag_dawson<br>Dawson's integral   |
| s15agc | 9  | nag_erfcx<br>Scaled complement of error function, $\operatorname{erfcx}(x)$   |
| s15ddc | 7  | nag_complex_erfc<br>Scaled complex complement of error function, $\exp(-z^2) \operatorname{erfc}(-iz)$                            |
| s17acc | 1  | nag_bessel_y0<br>Bessel function $Y_0(x)$   |
| s17adc | 1  | nag_bessel_y1<br>Bessel function $Y_1(x)$   |
| s17aec | 1  | nag_bessel_j0<br>Bessel function $J_0(x)$   |
| s17afc | 1  | nag_bessel_j1<br>Bessel function $J_1(x)$   |
| s17agc | 1  | nag_airy_ai<br>Airy function $\operatorname{Ai}(x)$   |
| s17ahc | 1  | nag_airy_bi<br>Airy function $\operatorname{Bi}(x)$   |
| s17ajc | 1  | nag_airy_ai_deriv<br>Airy function $\operatorname{Ai}'(x)$  |
| s17akc | 1  | nag_airy_bi_deriv<br>Airy function $\operatorname{Bi}'(x)$  |
| s17alc | 6  | nag_bessel_zeros<br>Zeros of Bessel functions $J_\alpha(x)$ , $J'_\alpha(x)$ , $Y_\alpha(x)$ or $Y'_\alpha(x)$                    |
| s17aqc | 23 | nag_bessel_y0_vector<br>Bessel function vectorized $Y_0(x)$   |
| s17arc | 23 | nag_bessel_y1_vector<br>Bessel function vectorized $Y_1(x)$   |
| s17asc | 23 | nag_bessel_j0_vector<br>Bessel function vectorized $J_0(x)$   |
| s17atc | 23 | nag_bessel_j1_vector<br>Bessel function vectorized $J_1(x)$   |
| s17auc | 23 | nag_airy_ai_vector<br>Airy function vectorized $\operatorname{Ai}(x)$   |
| s17avc | 23 | nag_airy_bi_vector<br>Airy function vectorized $\operatorname{Bi}(x)$   |
| s17awc | 23 | nag_airy_ai_deriv_vector<br>Derivatives of the Airy function, vectorized $\operatorname{Ai}'(x)$                                  |
| s17axc | 23 | nag_airy_bi_deriv_vector<br>Derivatives of the Airy function, vectorized $\operatorname{Bi}'(x)$                                  |
| s17dcc | 7  | nag_complex_bessel_y<br>Bessel functions $Y_{\nu+a}(z)$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$                  |
| s17dec | 7  | nag_complex_bessel_j<br>Bessel functions $J_{\nu+a}(z)$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$                  |
| s17dgc | 7  | nag_complex_airy_ai<br>Airy functions $\operatorname{Ai}(z)$ and $\operatorname{Ai}'(z)$ , complex $z$                            |
| s17dhc | 7  | nag_complex_airy_bi<br>Airy functions $\operatorname{Bi}(z)$ and $\operatorname{Bi}'(z)$ , complex $z$                            |
| s17dlc | 7  | nag_complex_hankel<br>Hankel functions $H_{\nu+a}^{(j)}(z)$ , $j = 1, 2$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$ |
| s18acc | 1  | nag_bessel_k0<br>Modified Bessel function $K_0(x)$  |
| s18adc | 1  | nag_bessel_k1<br>Modified Bessel function $K_1(x)$  |
| s18aec | 1  | nag_bessel_i0<br>Modified Bessel function $I_0(x)$  |

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|--------|----|---|
| s18afc | 1  | nag_bessel_i1<br>Modified Bessel function $I_1(x)$  |
| s18aqc | 23 | nag_bessel_k0_vector<br>Modified Bessel function vectorized $K_0(x)$  |
| s18arc | 23 | nag_bessel_k1_vector<br>Modified Bessel function vectorized $K_1(x)$  |
| s18asc | 23 | nag_bessel_i0_vector<br>Modified Bessel function vectorized $I_0(x)$  |
| s18atc | 23 | nag_bessel_i1_vector<br>Modified Bessel function vectorized $I_1(x)$  |
| s18ccc | 2  | nag_bessel_k0_scaled<br>Scaled modified Bessel function $e^x K_0(x)$  |
| s18cdc | 2  | nag_bessel_k1_scaled<br>Scaled modified Bessel function $e^x K_1(x)$  |
| s18cec | 2  | nag_bessel_i0_scaled<br>Scaled modified Bessel function $e^{- x } I_0(x)$   |
| s18cfc | 2  | nag_bessel_i1_scaled<br>Scaled modified Bessel function $e^{- x } I_1(x)$   |
| s18cqc | 23 | nag_bessel_k0_scaled_vector<br>Scaled modified Bessel function vectorized $e^x K_0(x)$  |
| s18crc | 23 | nag_bessel_k1_scaled_vector<br>Scaled modified Bessel function vectorized $e^x K_1(x)$  |
| s18csc | 23 | nag_bessel_i0_scaled_vector<br>Scaled modified Bessel function vectorized $e^{- x } I_0(x)$   |
| s18ctc | 23 | nag_bessel_i1_scaled_vector<br>Scaled modified Bessel function vectorized $e^{- x } I_1(x)$   |
| s18dcc | 7  | nag_complex_bessel_k<br>Modified Bessel functions $K_{\nu+a}(z)$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$   |
| s18dec | 7  | nag_complex_bessel_i<br>Modified Bessel functions $I_{\nu+a}(z)$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$   |
| s18ecc | 6  | nag_bessel_i_nu_scaled<br>Scaled modified Bessel function $e^{-x} I_{\nu/4}(x)$   |
| s18edc | 6  | nag_bessel_k_nu_scaled<br>Scaled modified Bessel function $e^x K_{\nu/4}(x)$  |
| s18eec | 6  | nag_bessel_i_nu<br>Modified Bessel function $I_{\nu/4}(x)$  |
| s18efc | 6  | nag_bessel_k_nu<br>Modified Bessel function $K_{\nu/4}(x)$  |
| s18egc | 6  | nag_bessel_k_alpha<br>Modified Bessel functions $K_{\alpha+n}(x)$ for real $x > 0$ , selected values of $\alpha \geq 0$ and $n = 0, 1, \dots, N$                          |
| s18ehc | 6  | nag_bessel_k_alpha_scaled<br>Scaled modified Bessel functions $e^x K_{\alpha+n}(x)$ for real $x > 0$ , selected values of $\alpha \geq 0$ and $n = 0, 1, \dots, N$        |
| s18ejc | 6  | nag_bessel_i_alpha<br>Modified Bessel functions $I_{\alpha+n-1}(x)$ or $I_{\alpha-n+1}(x)$ for real $x \neq 0$ , non-negative $\alpha < 1$ and $n = 1, 2, \dots,  N  + 1$ |
| s18ekc | 6  | nag_bessel_j_alpha<br>Bessel functions $J_{\alpha+n-1}(x)$ or $J_{\alpha-n+1}(x)$ for real $x \neq 0$ , non-negative $\alpha < 1$ and $n = 1, 2, \dots,  N  + 1$          |
| s18gkc | 7  | nag_complex_bessel_j_seq<br>Bessel function of the 1st kind $J_{\alpha \pm n}(z)$   |
| s19aac | 1  | nag_kelvin_ber<br>Kelvin function ber $x$   |
| s19abc | 1  | nag_kelvin_bei<br>Kelvin function bei $x$   |
| s19acc | 1  | nag_kelvin_ker<br>Kelvin function ker $x$   |

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| s19adc | 1  | nag_kelvin_kei<br>Kelvin function kei $x$   |
| s19anc | 23 | nag_kelvin_ber_vector<br>Kelvin function vectorized ber $x$   |
| s19apc | 23 | nag_kelvin_bei_vector<br>Kelvin function vectorized bei $x$   |
| s19aqc | 23 | nag_kelvin_ker_vector<br>Kelvin function vectorized ker $x$   |
| s19arc | 23 | nag_kelvin_kei_vector<br>Kelvin function vectorized kei $x$   |
| s20acc | 1  | nag_fresnel_s<br>Fresnel integral $S(x)$  |
| s20adc | 1  | nag_fresnel_c<br>Fresnel integral $C(x)$  |
| s20aqc | 23 | nag_fresnel_s_vector<br>Fresnel integral vectorized $S(x)$  |
| s20arc | 23 | nag_fresnel_c_vector<br>Fresnel integral vectorized $C(x)$  |
| s21bac | 1  | nag_elliptic_integral_rc<br>Degenerate symmetrised elliptic integral of 1st kind $R_C(x, y)$            |
| s21bbc | 1  | nag_elliptic_integral_rf<br>Symmetrised elliptic integral of 1st kind $R_F(x, y, z)$                    |
| s21bcc | 1  | nag_elliptic_integral_rd<br>Symmetrised elliptic integral of 2nd kind $R_D(x, y, z)$                    |
| s21bdc | 1  | nag_elliptic_integral_rj<br>Symmetrised elliptic integral of 3rd kind $R_J(x, y, z, r)$                 |
| s21bec | 9  | nag_elliptic_integral_F<br>Elliptic integral of 1st kind, Legendre form, $F(\phi   m)$                  |
| s21bfc | 9  | nag_elliptic_integral_E<br>Elliptic integral of 2nd kind, Legendre form, $E(\phi   m)$                  |
| s21bgc | 9  | nag_elliptic_integral_pi<br>Elliptic integral of 3rd kind, Legendre form, $\Pi(n; \phi   m)$            |
| s21bhc | 9  | nag_elliptic_integral_complete_K<br>Complete elliptic integral of 1st kind, Legendre form, $K(m)$       |
| s21bjc | 9  | nag_elliptic_integral_complete_E<br>Complete elliptic integral of 2nd kind, Legendre form, $E(m)$       |
| s21cac | 7  | nag_real_jacobian_elliptic<br>Jacobian elliptic functions sn, cn and dn of real argument                |
| s21cbc | 6  | nag_jacobian_elliptic<br>Jacobian elliptic functions sn, cn and dn of complex argument                  |
| s21ccc | 6  | nag_jacobian_theta<br>Jacobian theta functions with real arguments                                      |
| s21dac | 6  | nag_general_elliptic_integral_f<br>Elliptic integrals of the second kind with complex arguments         |
| s22aac | 6  | nag_legendre_p<br>Legendre and associated Legendre functions of the first kind with real arguments      |
| s22bac | 24 | nag_specfun_1fl_real<br>Real confluent hypergeometric function ${}_1F_1(a; b; x)$                       |
| s22bbc | 24 | nag_specfun_1fl_real_scaled<br>Real confluent hypergeometric function ${}_1F_1(a; b; x)$ in scaled form |
| s22bec | 24 | nag_specfun_2fl_real<br>Real Gauss hypergeometric function ${}_2F_1(a, b; c; x)$                        |
| s22bfc | 24 | nag_specfun_2fl_real_scaled<br>Real Gauss hypergeometric function ${}_2F_1(a, b; c; x)$ in scaled form. |
| s30aac | 9  | nag_bsm_price<br>Black–Scholes–Merton option pricing formula  |

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| s30abc | 9  | nag_bsm_greeks<br>Black–Scholes–Merton option pricing formula with Greeks  |
| s30bac | 9  | nag_lookback_fls_price<br>Floating-strike lookback option pricing formula in the Black-Scholes-Merton model              |
| s30bbc | 9  | nag_lookback_fls_greeks<br>Floating-strike lookback option pricing formula with Greeks in the Black-Scholes-Merton model |
| s30cac | 9  | nag_binary_con_price<br>Binary option, cash-or-nothing pricing formula   |
| s30cbc | 9  | nag_binary_con_greeks<br>Binary option, cash-or-nothing pricing formula with Greeks                                      |
| s30ccc | 9  | nag_binary_aon_price<br>Binary option, asset-or-nothing pricing formula  |
| s30cdc | 9  | nag_binary_aon_greeks<br>Binary option, asset-or-nothing pricing formula with Greeks                                     |
| s30fac | 9  | nag_barrier_std_price<br>Standard barrier option pricing formula   |
| s30jac | 9  | nag_jumpdiff_merton_price<br>Jump-diffusion, Merton’s model, option pricing formula                                      |
| s30jbc | 9  | nag_jumpdiff_merton_greeks<br>Jump-diffusion, Merton’s model, option pricing formula with Greeks                         |
| s30nac | 9  | nag_heston_price<br>Heston’s model option pricing formula  |
| s30nbc | 23 | nag_heston_greeks<br>Heston’s model option pricing formula with Greeks   |
| s30ncc | 24 | nag_heston_term<br>Heston’s model option pricing with term structure   |
| s30qcc | 9  | nag_amer_bs_price<br>American option, Bjerksund and Stensland pricing formula  |
| s30sac | 9  | nag_asian_geom_price<br>Asian option, geometric continuous average rate pricing formula                                  |
| s30sbc | 9  | nag_asian_geom_greeks<br>Asian option, geometric continuous average rate pricing formula with Greeks                     |

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