

# NAG Library Function Document

## nag\_elliptic\_integral\_pi (s21bgc)

### 1 Purpose

nag\_elliptic\_integral\_pi (s21bgc) returns a value of the classical (Legendre) form of the incomplete elliptic integral of the third kind.

### 2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_elliptic_integral_pi (double dn, double phi, double dm,
                                NagError *fail)
```

### 3 Description

nag\_elliptic\_integral\_pi (s21bgc) calculates an approximation to the integral

$$\Pi(n; \phi | m) = \int_0^\phi (1 - n \sin^2 \theta)^{-1} (1 - m \sin^2 \theta)^{-\frac{1}{2}} d\theta,$$

where  $0 \leq \phi \leq \frac{\pi}{2}$ ,  $m \sin^2 \phi \leq 1$ ,  $m$  and  $\sin \phi$  may not both equal one, and  $n \sin^2 \phi \neq 1$ .

The integral is computed using the symmetrised elliptic integrals of Carlson (Carlson (1979) and Carlson (1988)). The relevant identity is

$$\Pi(n; \phi | m) = \sin \phi R_F(q, r, 1) + \frac{1}{3} n \sin^3 \phi R_J(q, r, 1, s),$$

where  $q = \cos^2 \phi$ ,  $r = 1 - m \sin^2 \phi$ ,  $s = 1 - n \sin^2 \phi$ ,  $R_F$  is the Carlson symmetrised incomplete elliptic integral of the first kind (see nag\_elliptic\_integral\_rf (s21bbc)) and  $R_J$  is the Carlson symmetrised incomplete elliptic integral of the third kind (see nag\_elliptic\_integral\_rj (s21bdc)).

### 4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Carlson B C (1979) Computing elliptic integrals by duplication *Numerische Mathematik* **33** 1–16

Carlson B C (1988) A table of elliptic integrals of the third kind *Math. Comput.* **51** 267–280

### 5 Arguments

1:	<b>dn</b> – double	<i>Input</i>
2:	<b>phi</b> – double	<i>Input</i>
3:	<b>dm</b> – double	<i>Input</i>

*On entry:* the arguments  $n$ ,  $\phi$  and  $m$  of the function.

*Constraints:*

$0.0 \leq \mathbf{phi} \leq \frac{\pi}{2}$ ;  
 $\mathbf{dm} \times \sin^2(\mathbf{phi}) \leq 1.0$ ;  
 Only one of  $\sin(\mathbf{phi})$  and  $\mathbf{dm}$  may be 1.0;  
 $\mathbf{dn} \times \sin^2(\mathbf{phi}) \neq 1.0$ .

Note that  $\mathbf{dm} \times \sin^2(\mathbf{phi}) = 1.0$  is allowable, as long as  $\mathbf{dm} \neq 1.0$ .

4: **fail** – NagError \*

*Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 3.2.1.2 in the Essential Introduction for further information.

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 3.6.6 in the Essential Introduction for further information.

### NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 3.6.5 in the Essential Introduction for further information.

### NE\_REAL

On entry,  $\mathbf{phi} = \langle value \rangle$ .

Constraint:  $0 \leq \mathbf{phi} \leq (\pi/2)$ .

### NE\_REAL\_2

On entry,  $\mathbf{phi} = \langle value \rangle$  and  $\mathbf{dm} = \langle value \rangle$ ; the integral is undefined.

Constraint:  $\mathbf{dm} \times \sin^2(\mathbf{phi}) \leq 1.0$ .

On entry,  $\mathbf{phi} = \langle value \rangle$  and  $\mathbf{dn} = \langle value \rangle$ ; the integral is infinite.

Constraint:  $\mathbf{dn} \times \sin^2(\mathbf{phi}) \neq 1.0$ .

### NW\_INTEGRAL\_INFINITE

On entry,  $\sin(\mathbf{phi}) = 1$  and  $\mathbf{dm} = 1.0$ ; the integral is infinite.

## 7 Accuracy

In principle nag\_elliptic\_integral\_pi (s21bgc) is capable of producing full *machine precision*. However round-off errors in internal arithmetic will result in slight loss of accuracy. This loss should never be excessive as the algorithm does not involve any significant amplification of round-off error. It is reasonable to assume that the result is accurate to within a small multiple of the *machine precision*.

## 8 Parallelism and Performance

Not applicable.

## 9 Further Comments

You should consult the s Chapter Introduction, which shows the relationship between this function and the Carlson definitions of the elliptic integrals. In particular, the relationship between the argument-constraints for both forms becomes clear.

For more information on the algorithms used to compute  $R_F$  and  $R_J$ , see the function documents for nag\_elliptic\_integral\_rf (s21bbc) and nag\_elliptic\_integral\_rj (s21bdc), respectively.

If you wish to input a value of **phi** outside the range allowed by this function you should refer to Section 17.4 of Abramowitz and Stegun (1972) for useful identities.

## 10 Example

This example simply generates a small set of nonextreme arguments that are used with the function to produce the table of results.

### 10.1 Program Text

```

/* nag_elliptic_integral_pi (s21bgc) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 9, 2009.
 */
/* Pre-processor includes */
#include <stdio.h>
#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>
#include <nagx01.h>

int main(void)
{
    /*Integer scalar and array declarations */
    Integer exit_status = 0;
    Integer ix;
    /*Double scalar and array declarations */
    double dm, dn, p, phi, pi;
    NagError fail;

    INIT_FAIL(fail);

    printf("nag_elliptic_integral_pi (s21bgc) Example Program Results\n");
    printf("\n      dn      phi      dm      nag_elliptic_integral_pi\n\n");
    /*
     * nag_pi (x01aac)
     */
    pi = nag_pi;
    for (ix = 1; ix <= 3; ix++)
    {
        phi = ix*pi/6.00e0;
        dm = ix*0.250e0;
        dn = (pow((-1.00e0)), (ix+1))*ix*0.10e0;
        /*
         * nag_elliptic_integral_pi (s21bgc)
         * Elliptic integral of 3rd kind, Legendre form, Pi (n; phi |m)
         */
        p = nag_elliptic_integral_pi(dn, phi, dm, &fail);
        if (fail.code != NE_NOERROR)
        {
            printf("Error from nag_elliptic_integral_pi (s21bgc).\n%s\n",
                fail.message);
            exit_status = 1;
            goto END;
        }
        printf("%7.2f%7.2f%7.2f%12.4f\n", dn, phi, dm, p);
    }

    END:

    return exit_status;
}

```

## 10.2 Program Data

None.

## 10.3 Program Results

nag\_elliptic\_integral\_pi (s21bgc) Example Program Results

dn	phi	dm	nag_elliptic_integral_pi
0.10	0.52	0.25	0.5341
-0.20	1.05	0.50	1.0778
0.30	1.57	0.75	2.6568

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