NAG Library Function Document

nag_bessel_y0_vector (s17aqc)

1 Purpose

nag_bessel_y0_vector (s17aqc) returns an array of values of the Bessel function $Y_0(x)$.

2 Specification

```
#include <nag.h>
#include <nags.h>
```

```
void nag_bessel_y0_vector (Integer n, const double x[], double f[],
     Integer ivalid[], NagError *fail)
```

3 Description

nag_bessel_y0_vector (s17aqc) evaluates an approximation to the Bessel function of the second kind $Y_0(x_i)$ for an array of arguments x_i, for $i = 1, 2, \ldots, n$.

Note: $Y_0(x)$ is undefined for $x \leq 0$ and the function will fail for such arguments.

The function is based on four Chebyshev expansions:

For $0 < x \leq 8$,

$$Y_0(x) = \frac{2}{\pi} \ln x \sum_{r=0} a_r T_r(t) + \sum_{r=0} b_r T_r(t), \qquad \text{with } t = 2\left(\frac{x}{8}\right)^2 - 1.$$

For $x > 8$,

$$Y_0(x) = \sqrt{\frac{2}{\pi x}} \left\{ P_0(x) \sin\left(x - \frac{\pi}{4}\right) + Q_0(x) \cos\left(x - \frac{\pi}{4}\right) \right\}$$

where $P_0(x) = \sum_{r=0} c_r T_r(t)$,

and $Q_0(x) = \frac{8}{x} \sum_{r=0} d_r T_r(t), \text{with } t = 2\left(\frac{8}{x}\right)^2 - 1.$

For x near zero, $Y_0(x) \simeq \frac{2}{\pi}\left(\ln\left(\frac{x}{2}\right) + \gamma\right)$, where γ denotes Euler's constant. This approximation is used when x is sufficiently small for the result to be correct to ***machine precision***.

For very large x, it becomes impossible to provide results with any reasonable accuracy (see Section 7), hence the function fails. Such arguments contain insufficient information to determine the phase of oscillation of $Y_0(x)$; only the amplitude, $\sqrt{\frac{2}{\pi n}}$, can be determined and this is returned on failure. The range for which this occurs is roughly related to ***machine precision***; the function will fail if $x \gtrsim 1/$***machine precision*** (see the Users' Note for your implementation for details).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Clenshaw C W (1962) Chebyshev Series for Mathematical Functions *Mathematical tables* HMSO

5 Arguments

1: **n** – Integer *Input*

On entry: n, the number of points.

Constraint: $\mathbf{n} \geq 0$.

2: **x**[**n**] – const double *Input*

On entry: the argument x_i of the function, for $i = 1, 2, \ldots, \mathbf{n}$.

Constraint: $\mathbf{x}[i-1] > 0.0$, for $i = 1, 2, \ldots, \mathbf{n}$.

3: **f**[**n**] – double *Output*

On exit: $Y_0(x_i)$, the function values.

4: **ivalid**[**n**] – Integer *Output*

On exit: **ivalid**$[i-1]$ contains the error code for x_i, for $i = 1, 2, \ldots, \mathbf{n}$.

ivalid$[i-1] = 0$
No error.

ivalid$[i-1] = 1$

On entry, x_i is too large. $\mathbf{f}[i-1]$ contains the amplitude of the Y_0 oscillation, $\sqrt{\frac{2}{\pi x_i}}$.

ivalid$[i-1] = 2$

On entry, $x_i \leq 0.0$, Y_0 is undefined. $\mathbf{f}[i-1]$ contains 0.0.

5: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: $\mathbf{n} \geq 0$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

NW_IVALID

On entry, at least one value of **x** was invalid.
Check **ivalid** for more information.

7 Accuracy

Let δ be the relative error in the argument and E be the absolute error in the result. (Since $Y_0(x)$ oscillates about zero, absolute error and not relative error is significant, except for very small x.)

If δ is somewhat larger than the machine representation error (e.g., if δ is due to data errors etc.), then E and δ are approximately related by

$$E \simeq |xY_1(x)|\delta$$

(provided E is also within machine bounds). Figure 1 displays the behaviour of the amplification factor $|xY_1(x)|$.

However, if δ is of the same order as the machine representation errors, then rounding errors could make E slightly larger than the above relation predicts.

For very small x, the errors are essentially independent of δ and the function should provide relative accuracy bounded by the ***machine precision***.

For very large x, the above relation ceases to apply. In this region, $Y_0(x) \simeq \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{\pi}{4}\right)$. The amplitude $\sqrt{\frac{2}{\pi x}}$ can be calculated with reasonable accuracy for all x, but $\sin\left(x - \frac{\pi}{4}\right)$ cannot. If $x - \frac{\pi}{4}$ is written as $2N\pi + \theta$ where N is an integer and $0 \leq \theta < 2\pi$, then $\sin\left(x - \frac{\pi}{4}\right)$ is determined by θ only. If $x \gtrsim \delta^{-1}$, θ cannot be determined with any accuracy at all. Thus if x is greater than, or of the order of the inverse of ***machine precision***, it is impossible to calculate the phase of $Y_0(x)$ and the function must fail.

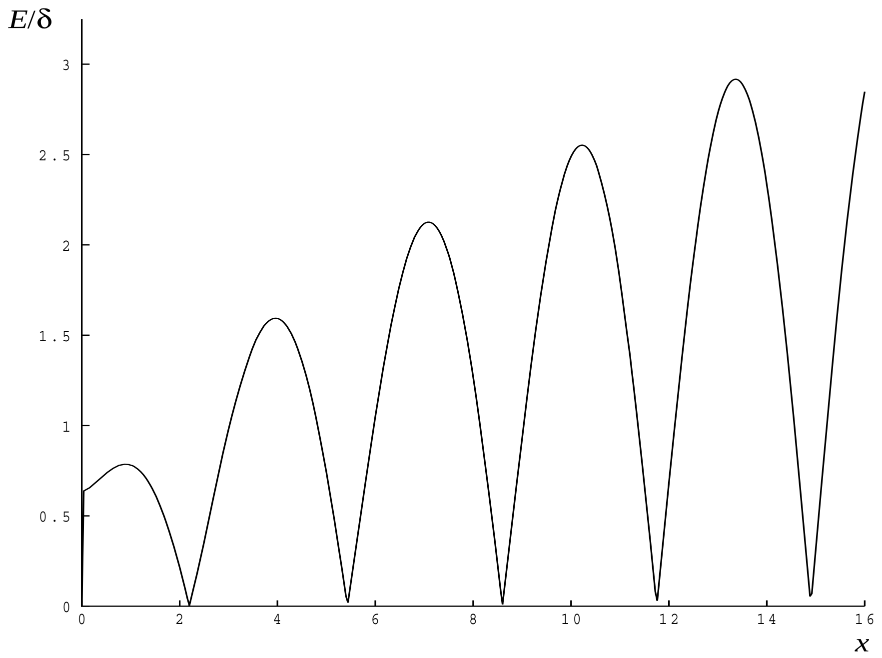

Figure 1

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of **x** from a file, evaluates the function at each value of x_i and prints the results.

10.1 Program Text

```
/* nag_bessel_y0_vector (s17aqc) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 23, 2011.
 */
#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Integer  exit_status = 0;
  Integer  i, n;
  double   *f = 0, *x = 0;
  Integer  *ivalid = 0;
  NagError fail;

  INIT_FAIL(fail);

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif

  printf("nag_bessel_y0_vector (s17aqc) Example Program Results\n");
  printf("\n");
  printf("     x           f          ivalid\n");
  printf("\n");
#ifdef _WIN32
  scanf_s("%"NAG_IFMT"", &n);
#else
  scanf("%"NAG_IFMT"", &n);
#endif
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif

  /* Allocate memory */
  if (!(x = NAG_ALLOC(n, double)) ||
      !(f = NAG_ALLOC(n, double)) ||
      !(ivalid = NAG_ALLOC(n, Integer)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  for (i=0; i<n; i++)
#ifdef _WIN32
    scanf_s("%lf", &x[i]);
#else
```

```
    scanf("%lf", &x[i]);
#endif
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif

  /* nag_bessel_y0_vector (s17aqc).
   * Bessel function Y_0(x)
   */
  nag_bessel_y0_vector(n, x, f, ivalid, &fail);
  if (fail.code!=NE_NOERROR && fail.code!=NW_IVALID)
    {
      printf("Error from nag_bessel_y0_vector (s17aqc).\n%s\n",
             fail.message);
      exit_status = 1;
      goto END;
    }

  for (i=0; i<n; i++)
    printf(" %11.3e %11.3e %4"NAG_IFMT"\n", x[i], f[i], ivalid[i]);

 END:
  NAG_FREE(f);
  NAG_FREE(x);
  NAG_FREE(ivalid);

  return exit_status;
}
```

10.2 Program Data

```
nag_bessel_y0_vector (s17aqc) Example Program Data

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0.5 1.0 3.0 6.0 8.0 10.0 1000.0
```

10.3 Program Results

```
nag_bessel_y0_vector (s17aqc) Example Program Results

     x           f          ivalid

   5.000e-01  -4.445e-01    0
   1.000e+00   8.826e-02    0
   3.000e+00   3.769e-01    0
   6.000e+00  -2.882e-01    0
   8.000e+00   2.235e-01    0
   1.000e+01   5.567e-02    0
   1.000e+03   4.716e-03    0
```