NAG Library Function Document

nag_erf (s15aec)

1 Purpose

nag_erf (s15aec) returns the value of the error function $\mathrm{erf}(x)$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_erf (double x)
```

3 Description

nag_erf (s15aec) calculates an approximate value for the error function

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt = 1 - \mathrm{erfc}(x).$$

Let $\hat{x}$ be the root of the equation $\mathrm{erfc}(x) - \mathrm{erf}(x) = 0$ (then $\hat{x} \approx 0.46875$). For $|x| \leq \hat{x}$ the value of $\mathrm{erf}(x)$ is based on the following rational Chebyshev expansion for $\mathrm{erf}(x)$:

$$\mathrm{erf}(x) \approx xR_{\ell,m}\left(x^2\right),$$

where $R_{\ell,m}$ denotes a rational function of degree ℓ in the numerator and m in the denominator.

For $|x| > \hat{x}$ the value of $\mathrm{erf}(x)$ is based on a rational Chebyshev expansion for $\mathrm{erfc}(x)$: for $\hat{x} < |x| \leq 4$ the value is based on the expansion

$$\mathrm{erfc}(x) \approx e^{x^2}R_{\ell,m}(x);$$

and for $|x| > 4$ it is based on the expansion

$$\mathrm{erfc}(x) \approx \frac{e^{x^2}}{x}\left(\frac{1}{\sqrt{\pi}} + \frac{1}{x^2}R_{\ell,m}\left(1/x^2\right)\right).$$

For each expansion, the specific values of ℓ and m are selected to be minimal such that the maximum relative error in the expansion is of the order 10^{-d}, where d is the maximum number of decimal digits that can be accurately represented for the particular implementation (see nag_decimal_digits (X02BEC)).

For $|x| \geq x_{\mathrm{hi}}$ there is a danger of setting underflow in $\mathrm{erfc}(x)$ (the value of x_{hi} is given in the Users' Note for your implementation). For $x \geq x_{\mathrm{hi}}$, nag_erf (s15aec) returns $\mathrm{erf}(x) = 1$; for $x \leq -x_{\mathrm{hi}}$ it returns $\mathrm{erf}(x) = -1$.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Cody W J (1969) Rational Chebyshev approximations for the error function *Math.Comp.* **23** 631–637

5 Arguments

1: **x** – double *Input*

On entry: the argument x of the function.

6 Error Indicators and Warnings

None.

7 Accuracy

See Section 7 in nag_erfc (s15adc).

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
/* nag_erf (s15aec) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 1, 1990.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Integer exit_status = 0;
  double  x, y;

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif
  printf("nag_erf (s15aec) Example Program Results\n");
  printf("     x           y\n");
#ifdef _WIN32
  while (scanf_s("%lf", &x) != EOF)
#else
  while (scanf("%lf", &x) != EOF)
#endif
    {
      /* nag_erf (s15aec).
       * Error function erf(x)
       */
      y = nag_erf(x);
      printf("%12.3e%12.3e\n", x, y);
    }

  return exit_status;
}
```

10.2 Program Data

```
nag_erf (s15aec) Example Program Data
            -6.0
            -4.5
            -1.0
             1.0
             4.5
             6.0
```

10.3 Program Results

```
nag_erf (s15aec) Example Program Results
     x           y
  -6.000e+00  -1.000e+00
  -4.500e+00  -1.000e-00
  -1.000e+00  -8.427e-01
   1.000e+00   8.427e-01
   4.500e+00   1.000e-00
   6.000e+00   1.000e+00
```