NAG Library Function Document

nag_scaled_log_gamma (s14ahc)

1 Purpose

nag_scaled_log_gamma (s14ahc) returns the value of $\ln G(x)$, the scaled logarithm of the gamma function $\Gamma(x)$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_scaled_log_gamma (double x, NagError *fail)
```

3 Description

nag_scaled_log_gamma (s14ahc) calculates an approximate value for $\ln G(x)$, where $G(x) = \Gamma(x+1)/\left(\frac{x}{e}\right)^x$. This is a variant of the $\ln\Gamma(x)$ function (see also nag_log_gamma (s14abc)), which avoids rounding problems for very large arguments by computing $\ln\Gamma(x)$ with the Stirling approximation factored out.

For $0 < x < 15$, $\ln G(x) = \ln\Gamma(x+1) - x\ln x + x$;

and for $15 \leq x$, $\ln G(x) = \frac{1}{2}\ln x + \ln\left(\sqrt{2\pi}\right) + \frac{1}{x}R(1/x^2)$, where R is a suitable Remez approximation.

For $x \leq 0.0$, the value $\ln G(x)$ is undefined; nag_scaled_log_gamma (s14ahc) returns zero and exits with **fail**.**code** = NE_REAL_ARG_LE.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: **x** – double *Input*

On entry: the argument x of the function.

Constraint: **x** > 0.0.

2: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

NE_REAL_ARG_LE

On entry, $\mathbf{x} = \langle value \rangle$.
Constraint: $\mathbf{x} > 0.0$.

7 Accuracy

nag_scaled_log_gamma (s14ahc) has been designed to produce full relative accuracy for all input arguments. Empirical results obtained by comparing with multiprecision software confirm this.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
/* nag_scaled_log_gamma (s14ahc) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 9, 2009.
 */
/* Pre-processor includes */
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  /*Integer scalar and array declarations */
  Integer  exit_status = 0;
  /*Double scalar and array declarations */
  double   x, y;
  NagError fail;

  INIT_FAIL(fail);

  printf("nag_scaled_log_gamma (s14ahc) Example Program Results\n");
  /* Skip heading in data file*/
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif
  printf("\n%s\n\n", "       x          y");
```

```
#ifdef _WIN32
  while (scanf_s("%lf%*[^\n] ", &x) != EOF)
#else
  while (scanf("%lf%*[^\n] ", &x) != EOF)
#endif
    {
      /*
       * nag_scaled_log_gamma (s14ahc)
       * Scaled logarithm of Gamma function, G(x)
       */
      y = nag_scaled_log_gamma(x, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_scaled_log_gamma (s14ahc) %s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }
      printf("%14.5e%14.5e\n", x, y);
    }

 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_scaled_log_gamma (s14ahc) Example Program Data
             1.0
             1.25
             1.5
             1.75
             2.0
             5.0
             10.0
             20.0
             1000.0
```

10.3 Program Results

```
nag_scaled_log_gamma (s14ahc) Example Program Results

       x          y

  1.00000e+00  1.00000e+00
  1.25000e+00  1.09594e+00
  1.50000e+00  1.17649e+00
  1.75000e+00  1.24589e+00
  2.00000e+00  1.30685e+00
  5.00000e+00  1.74030e+00
  1.00000e+01  2.07856e+00
  2.00000e+01  2.42097e+00
  1.00000e+03  4.37290e+00
```

Example Program
Returns the Value of *lnG*(*x*), the Scaled Logarithm of the Gamma Function Γ(*x*)