NAG Library Function Document

nag_log_gamma (s14abc)

1 Purpose

nag_log_gamma (s14abc) returns the value of the logarithm of the gamma function, $\ln \Gamma(x)$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_log_gamma (double x, NagError *fail)
```

3 Description

nag_log_gamma (s14abc) calculates an approximate value for $\ln \Gamma(x)$. It is based on rational Chebyshev expansions.

Denote by $R^i_{n,m}(x) = P^i_n(x)/Q^i_m(x)$ a ratio of polynomials of degree n in the numerator and m in the denominator. Then:

for $0 < x \leq 1/2$,

$$\ln \Gamma(x) \approx -\ln(x) + xR^1_{n,m}(x+1);$$

for $1/2 < x \leq 3/2$,

$$\ln \Gamma(x) \approx (x-1)R^1_{n,m}(x);$$

for $3/2 < x \leq 4$,

$$\ln \Gamma(x) \approx (x-2)R^2_{n,m}(x);$$

for $4 < x \leq 12$,

$$\ln \Gamma(x) \approx R^3_{n,m}(x);$$

and for $12 < x$,

$$\ln \Gamma(x) \approx \left(x - \frac{1}{2}\right)\ln(x) - x + \ln\left(\sqrt{2\pi}\right) + \frac{1}{x}R^4_{n,m}\left(1/x^2\right). \quad (1)$$

For each expansion, the specific values of n and m are selected to be minimal such that the maximum relative error in the expansion is of the order 10^{-d}, where d is the maximum number of decimal digits that can be accurately represented for the particular implementation (see nag_decimal_digits (X02BEC)).

Let ϵ denote ***machine precision*** and let x_{huge} denote the largest positive model number (see nag_real_largest_number (X02ALC)). For $x < 0.0$ the value $\ln \Gamma(x)$ is not defined; nag_log_gamma (s14abc) returns zero and exits with **fail.code** = NE_REAL_ARG_LE. It also exits with **fail.code** = NE_REAL_ARG_LE when $x = 0.0$, and in this case the value x_{huge} is returned. For x in the interval $(0.0, \epsilon]$, the function $\ln \Gamma(x) = -\ln(x)$ to machine accuracy.

Now denote by x_{big} the largest allowable argument for $\ln \Gamma(x)$ on the machine. For $\left(x_{\text{big}}\right)^{1/4} < x \leq x_{\text{big}}$ the $R^4_{n,m}(1/x^2)$ term in Equation (1) is negligible. For $x > x_{\text{big}}$ there is a danger of setting overflow, and so nag_log_gamma (s14abc) exits with **fail.code** = NE_REAL_ARG_GT and returns x_{huge}. The value of x_{big} is given in the Users' Note for your implementation.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Cody W J and Hillstrom K E (1967) Chebyshev approximations for the natural logarithm of the gamma function *Math.Comp.* **21** 198–203

5 Arguments

1: **x** – double *Input*

On entry: the argument x of the function.

Constraint: $\mathbf{x} > 0.0$.

2: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

NE_REAL_ARG_GT

On entry, $\mathbf{x} = \langle value \rangle$.
Constraint: $\mathbf{x} \leq x_{\mathrm{big}}$.

NE_REAL_ARG_LE

On entry, $\mathbf{x} = \langle value \rangle$.
Constraint: $\mathbf{x} > 0.0$.

7 Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively, and E be the absolute error in the result.

If δ is somewhat larger than ***machine precision***, then

$$E \simeq |x \times \Psi(x)|\delta \quad \text{and} \quad \epsilon \simeq \left|\frac{x \times \Psi(x)}{\ln \Gamma(x)}\right|\delta$$

where $\Psi(x)$ is the digamma function $\frac{\Gamma'(x)}{\Gamma(x)}$. Figure 1 and Figure 2 show the behaviour of these error amplification factors.

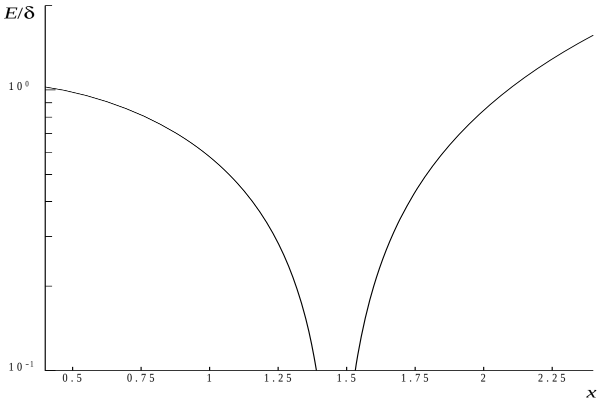

Figure 1

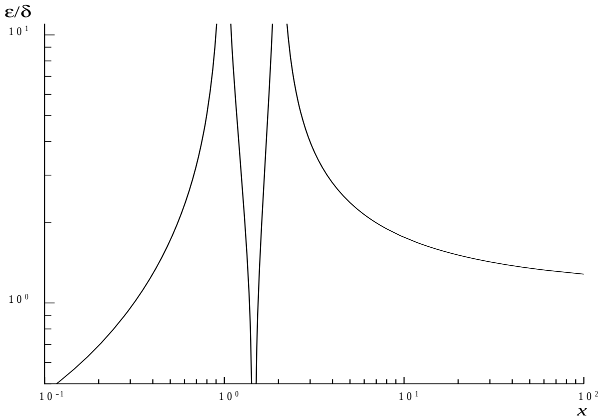

Figure 2

These show that relative error can be controlled, since except near $x = 1$ or 2 relative error is attenuated by the function or at least is not greatly amplified.

For large x, $\epsilon \simeq \left(1 + \frac{1}{\ln x}\right)\delta$ and for small x, $\epsilon \simeq \frac{1}{\ln x}\delta$.

The function $\ln \Gamma(x)$ has zeros at $x = 1$ and 2 and hence relative accuracy is not maintainable near those points. However absolute accuracy can still be provided near those zeros as is shown above.

If however, δ is of the order of ***machine precision***, then rounding errors in the function's internal arithmetic may result in errors which are slightly larger than those predicted by the equalities. It should be noted that even in areas where strong attenuation of errors is predicted the relative precision is bounded by the effective machine precision.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
/* nag_log_gamma (s14abc) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Integer  exit_status = 0;
  double   x, y;
  NagError fail;

  INIT_FAIL(fail);

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif
  printf("nag_log_gamma (s14abc) Example Program Results\n");
  printf("     x           y\n");
#ifdef _WIN32
  while (scanf_s("%lf", &x) != EOF)
#else
  while (scanf("%lf", &x) != EOF)
#endif
    {
      /* nag_log_gamma (s14abc).
       * Log Gamma function ln(Gamma(x))
       */
      y = nag_log_gamma(x, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_log_gamma (s14abc).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }
      printf("%12.3e%12.3e\n", x, y);
    }

 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_log_gamma (s14abc) Example Program Data
             1.0
             1.25
             1.5
             1.75
             2.0
             5.0
             10.0
             20.0
             1000.0
```

10.3 Program Results

```
nag_log_gamma (s14abc) Example Program Results
     x           y
   1.000e+00   0.000e+00
   1.250e+00  -9.827e-02
   1.500e+00  -1.208e-01
   1.750e+00  -8.440e-02
   2.000e+00   0.000e+00
   5.000e+00   3.178e+00
   1.000e+01   1.280e+01
   2.000e+01   3.934e+01
   1.000e+03   5.905e+03
```

Example Program
Returned Values for the Logarithm of the Gamma Function, $ln\Gamma(x)$